

NOTE: The theory in this application note is still applicable, but some of the products referenced may be discontinued.

# AN139A

## Understanding Transistor Response Parameters

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This note explains high-frequency transistor response parameters and discusses their interdependence. Useful nomograms are given for determining  $h_{fe}$ ,  $f_T$ ,  $f_{\alpha e}$ ,  $f_{max}$ , and many other parameters.

### ABSTRACT

The range of frequencies over which a transistor performs a useful circuit function is limited by inherent parameters. Manufacturers' data sheets often specify only one or two of these parameters, and questions concerning others often arise. Therefore, a clear understanding of these parameters is of value in attempting to answer such questions from the data given.

### PARAMETER CHARACTERISTICS AND INTERRELATIONSHIPS

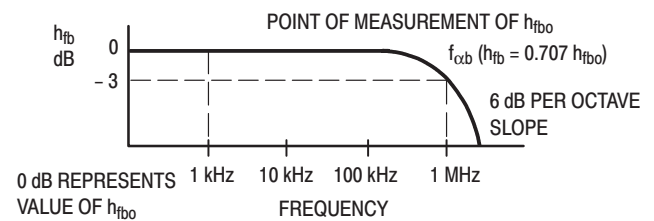
One parameter is  $h_{fb}$  (alpha, the common base ac short-circuit forward current gain). As frequency is increased,  $h_{fb}$  remains approximately equal to  $h_{fb0}$  (the value of  $h_{fb}$  at 1 kHz). After the upper frequency is reached,  $h_{fb}$  begins to decrease rapidly.

The frequency at which a significant decrease in  $h_{fb}$  occurs provides a basis for comparison of the expected high frequency performance of different transistors. The common base current gain cutoff frequency,  $f_{\alpha b}$ , is defined as that frequency at which  $h_{fb}$  is 3 dB below  $h_{fb0}$ . Expressed in magnitude,  $h_{fb}$  at  $f_{\alpha b}$ , is 70.7 percent of  $h_{fb0}$ . Power gains, current gains, and voltage gains for a few common decibel values are found in Table 1. A curve of  $h_{fb}$  versus frequency for a transistor with an  $f_{\alpha b}$  of 1 MHz is shown in Figure 1.

This curve has the following significant characteristics: (1) at frequencies below  $f_{\alpha b}$ ,  $h_{fb}$  is nearly constant and approximately equal to  $h_{fb0}$ , (2)  $h_{fb}$  begins to decrease significantly in the region of  $f_{\alpha b}$ , (3) above  $f_{\alpha b}$ , the rate of decrease in  $h_{fb}$  with increasing frequency approaches 6 dB per octave in the limit.

The curve of common base current gain versus frequency for any transistor has these characteristics, and the same general appearance as the curve of Figure 1.

The common emitter parameter which corresponds to  $f_{\alpha b}$ , is  $f_{\alpha e}$ , the common emitter current gain cutoff frequency.



**Figure 1. Graph Represents a Curve of a Common Base Current Gain Plotted Against Frequency Variations**

\* Circuit diagrams are included as a means of illustrating typical semiconductor applications, consequently, complete information sufficient for construction purposes is not necessarily given. The information in this application note has been carefully checked, and is believed to be entirely reliable. However, no responsibility is assumed for inaccuracies. Furthermore, such information does not convey to the purchaser of the semiconductor devices described any license under the patent rights of Motorola Inc. or others.

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GLOSSARY OF SYMBOLS

Symbol	Definition
$h_{fb}$	Common base ac forward current gain (alpha)
$h_{fbo}$	Value of $h_{fb}$ at 1 kHz
$h_{fe}$	Common emitter ac forward current gain (beta)
$h_{feo}$	Value of $h_{fe}$ at 1 kHz
$f_{\alpha b}$	Common base current gain cutoff frequency. Frequency at which $h_{fb}$ has decreased to a value 3 dB below $h_{fbo}$ . ( $h_{fb} = 0.707 h_{fbo}$ )
$f_{\alpha e}$	Common emitter current gain cutoff frequency. Frequency at which $h_{fe}$ has decreased to a value of 3 dB below $h_{feo}$ ( $h_{fe} = 0.707 h_{feo}$ )
$f_T$	Gain bandwidth product. Frequency at which $h_{fe} = 1$ (0 dB)
$G_{pe}$	Common emitter power gain
$f_{max}$	Maximum frequency of oscillation. Frequency at which $G_{pe} = 1$ (0 dB)
$K_{\theta}$	Excess phase shifter factor. Factor which is a function of excess phase shift of current in the base of a transistor.

By definition,  $f_{\alpha e}$  is the frequency at which  $h_{fe}$  (beta, the common emitter of ac short-circuit current gain), has decreased 3 dB below  $h_{feo}$  (the value of  $h_{fe}$  at 1 kHz). A typical curve of  $h_{fe}$  versus frequency for a transistor with an  $f_{\alpha e}$  of 100 kHz is shown in Figure 2.

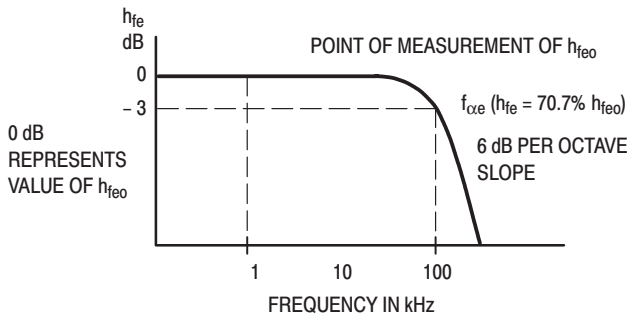


Figure 2. Common Emitter Current Gain is Plotted Against Frequency in the Curve Shown

This curve also has the significant characteristics listed for Figure 1. These characteristics allow such a curve to be constructed for a particular transistor by knowing only  $h_{feo}$  and  $f_{\alpha e}$ . From the curve,  $h_{fe}$  at any frequency could be determined. Furthermore, if  $f_{\alpha e}$  is not known, a curve could also be constructed if  $h_{feo}$  and  $h_{fe}$  at any frequency above  $f_{\alpha e}$  were known. Thus to determine  $h_{fe}$  at any frequency, it is necessary to know only  $h_{feo}$  and either  $f_{\alpha e}$  or  $h_{fe}$  at some frequency  $f$ , where  $f$  is greater than  $f_{\alpha e}$ .

Sometimes  $h_{feo}$  is needed and only  $h_{fbo}$  is given, or vice versa. The quantities  $h_{fbo}$  and  $h_{feo}$  are related by the following:

$$h_{feo} = \frac{h_{fbo}}{1 - h_{fbo}} \tag{1}$$

$$h_{fbo} = \frac{h_{feo}}{h_{feo} + 1} \tag{2}$$

Equations 1 and 2 are plotted in Figure 3. To further facilitate computations, the low frequency current gain scales of Figures 7–8 contain both an  $h_{fbo}$  and an  $h_{feo}$  scale, and may be entered with a knowledge of either quantity.

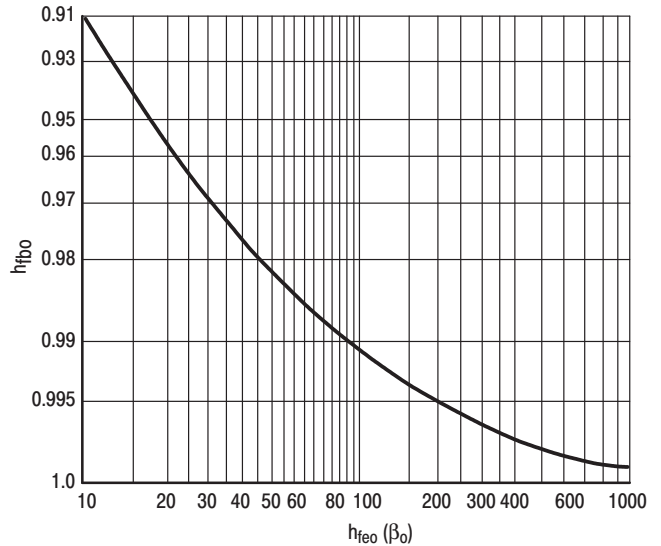


Figure 3. The Relationship Between  $h_{feo}$  and  $h_{fbo}$  is given by the Graph Shown

RELATIONSHIPS BETWEEN  $f_{\alpha e}$  AND  $f_{\alpha b}$

Suppose two transistors are considered for a particular application where performance at high frequencies is of interest. The data sheets are compared and it is discovered that one specifies  $f_{\alpha b}$  and the other  $f_{\alpha e}$ . What preliminary comparisons can be made from this without making any laboratory measurements?

Phillips<sup>1</sup> gives a discussion of the relationships between  $f_{\alpha e}$  and  $f_{\alpha b}$  with the following result:

$$f_{\alpha e} = K_{\theta} (1 - h_{fbo}) f_{\alpha b} \tag{3}$$

where  $K_{\theta}$  is a function of excess phase shift in the base region and has some value between 0.5 and 1.0.

Most transistors have a  $K_{\theta}$  in the 0.8 to 1.0 range. Alloy transistors have a  $K_{\theta}$  of 0.82.

The nomograms provide solutions for values of  $K_{\theta}$  of 0.9 and 0.8. If more specific information on  $K_{\theta}$  is not available, a value of 0.8 is recommended.

The quantity  $f_{\alpha e}$  is normally a much lower frequency than  $f_{\alpha b}$  for the same transistor. For example, consider the Motorola 2N1141 germanium transistor. The data sheets give typical values  $f_{\alpha b} = 1,000$  MHz and  $h_{fbo} = 0.98$ . Substituting in Equation 3 yields  $f_{\alpha e} = 0.80 (1 - 0.98) 1,000 = 16$  MHz.

This result is in approximate agreement with the  $h_{fe}$  versus frequency curve of the manufacturer's 2N1141 data sheet.

For the practical application of Equation 3, refer to Figure 7. When any two of the quantities  $f_{\alpha e}$ ,  $f_{\alpha b}$ ,  $h_{fbo}$ , or  $h_{feo}$  are known, use the nomograms to find the third quantity.

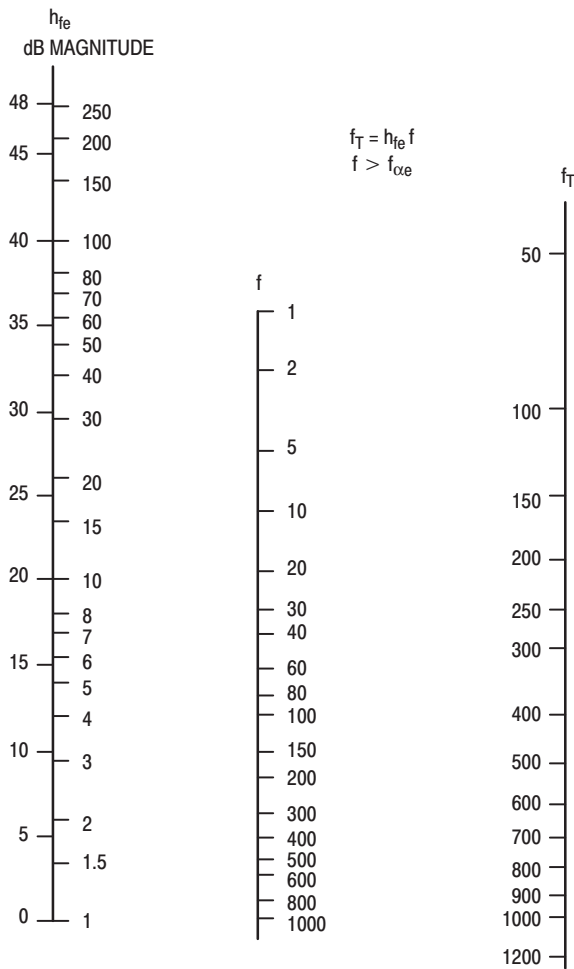
**Table 1. Conversion Table for Power, Voltage, and Current Ratios into Decibels**

dB	Power Ratio	Voltage or Current Ratio	dB	Power Ratio	Voltage or Current Ratio
0	1.00	1.00	10	10.0	3.2
0.5	1.12	1.06	15	31.5	5.6
1.0	1.26	1.12	20	100	10
1.5	1.41	1.19	25	316	18
2.0	1.58	1.26	30	1,000	32
3.0	2.00	1.41	40	10,000	100
4.0	2.51	1.58	50	10 <sup>5</sup>	316
5.0	3.16	1.78	60	10 <sup>6</sup>	1,000
6.0	3.98	2.00			
7.0	5.01	2.24			
8.0	6.31	2.51			
9.0	7.94	2.82			

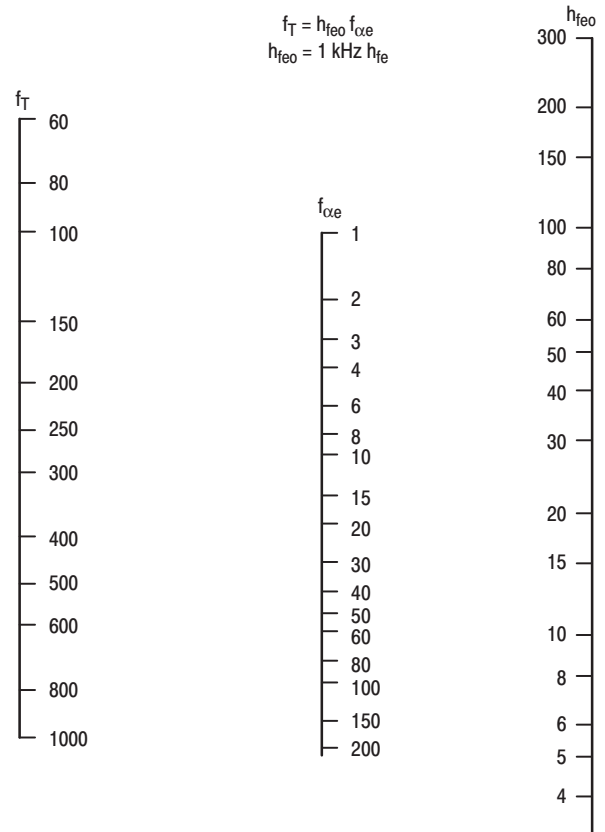
A common high-frequency parameter is  $f_T$ , the gain bandwidth product and is defined as that frequency at which  $h_{fe} = 1$  (0 dB).

The value  $f_T$  is sometimes specified indirectly on high-frequency transistor data sheets. This is done by specifying  $h_{fe}$  at some frequency above  $f_{\alpha e}$ , thus  $f_T$  is then obtained by multiplying the magnitude of  $h_{fe}$  by the frequency of measurement. This relationship arises from the 6 dB per octave characteristic of the  $h_{fe}$  versus frequency curve above  $f_{\alpha e}$ . Since 6 dB represents a current gain magnitude of 2,  $h_{fe}$  is halved each time frequency is doubled, and vice versa. Therefore, the product of  $h_{fe}$  and frequency of the sloping portion of the curve yields  $f_T$ .

For example, consider the Motorola 2N2218 silicon annular transistor. The data sheet gives a typical  $h_{fe}$  of 4.0 at 100 MHz. Multiplication of  $h_{fe}$  times the frequency of measurement yields  $f_T = 4.0 \times 100 = 400$  MHz. This is in agreement with the data sheet which specifies a typical  $f_T$  of 400 MHz.

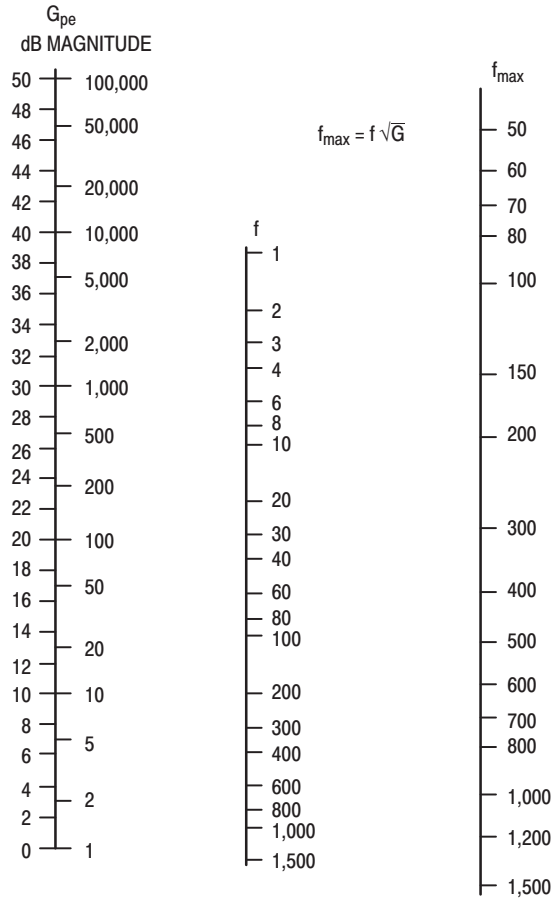


**Figure 4. This Nomogram is Useful in Finding  $h_{fe}$ , when a Frequency  $f > f_{\alpha e}$**



**Figure 5. The Quantity  $f_T$  is Found from this Nomogram Once  $f_{\alpha e}$  and  $h_{fe0}$  are Known**

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**Figure 6. Maximum Frequency is Found from this Nomogram Knowing the Frequency and Power Gain**

The parameter  $f_T$  is also equal to the product of  $h_{fe0}$  and  $f_{\alpha e}$ , expressed by

$$f_T = h_{fe0} \times f_{\alpha e} \quad (4)$$

with  $h_{fe0}$  known, Equation 4 provides a simple means of finding  $f_{\alpha e}$  when  $f_T$  is known or vice versa. (See Figure 5.)

Phillips also develops the following relationship between  $f_{\alpha b}$  and  $f_T$ :

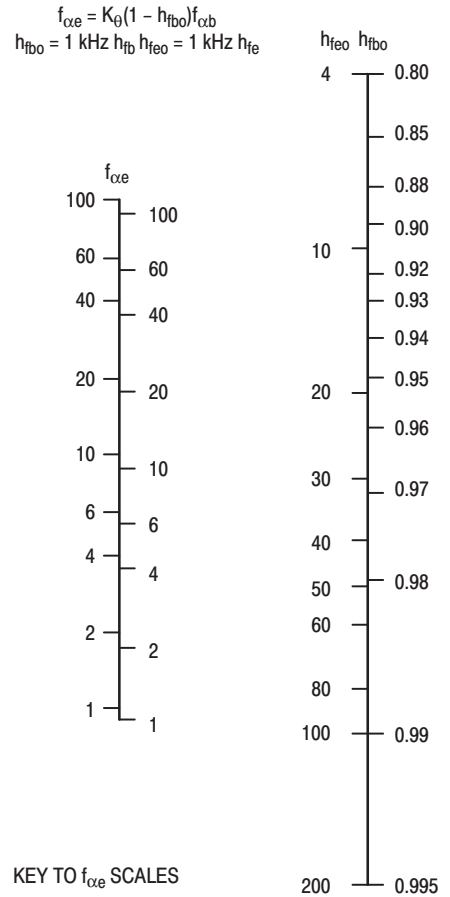
$$f_T = K_{\theta} h_{fb0} f_{\alpha b} \quad (5)$$

where  $K_{\theta}$  is the same quantity as in Equation 3. Notice that since  $K_{\theta}$  lies between 0.5 and 1.0, the  $f_T$  of a transistor is approximately equal to or slightly less than its  $f_{\alpha b}$ . (See Figure 8.)

## RULES FOR DETERMINING $h_{fe}$

The following rules summarize how to determine  $h_{fe}$  at some frequency  $f$ :

- Rule 1: When  $f < f_{\alpha e}$ ,  $h_{fe} \approx h_{fe0}$
- Rule 2: When  $f \approx f_{\alpha e}$ ,  $h_{fe} \approx 0.7 h_{fe0}$
- Rule 3: When  $f > f_{\alpha e}$ , consider  $h_{fe}$  to be decreasing at 6 dB per octave at frequency  $f$  and use Figure 4 to find  $h_{fe}$ .
- Rule 4: (A) If  $h_{fb0}$  not  $h_{fe0}$  is known, use Figure 3 to find  $h_{fe0}$ .



**Figure 7. Once  $f_{\alpha b}$  is Known this Nomogram is Used to Find  $f_{\alpha e}$**

(B) If  $h_{fe0}$  and  $f_{\alpha e}$  are known, use Figure 5 to find  $f_T$ . Use Figure 8 to find  $f_T$  if  $f_{\alpha b}$  is known.

(C) If  $f_T$  is known, use Figure 5 to find  $f_{\alpha e}$ . Use Figure 7 to find  $f_{\alpha b}$  if  $f_{\alpha e}$  is known.

Though common emitter current gain is equal to 1 at  $f_T$ , there may still be considerable power gain at  $f_T$  due to different input and output impedance levels. Thus,  $f_T$  is not necessarily the highest useful frequency of operation of a transistor, and an additional parameter, the maximum frequency of oscillation ( $f_{max}$ ), is sometimes encountered. The term  $f_{max}$  is the frequency at which common emitter power gain is equal to 1, and is related to  $f_T$  by

$$f_{max} \approx \sqrt{\frac{f_T}{8\pi r'_b C_c}} \quad (6)$$

where  $r'_b$  is the base resistance and  $C_c$  is the collector capacitance.

A plot of common emitter power gain versus frequency also has the characteristics shown in Figure 1. This leads to another gain bandwidth product

$$f_{max} \approx f \sqrt{\text{Power Gain}} \quad (7)$$

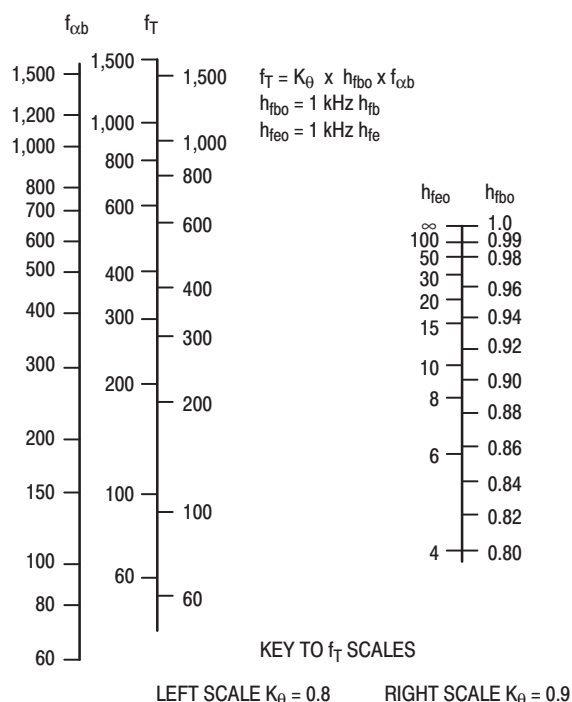
where  $f$  is the frequency of measurement and power gain is expressed in magnitude not in decibels. Hence,  $f_{max}$  may be found by measuring power gain at some frequency

on the 6 dB per octave portion of the power gain versus frequency curve, and multiplying the square root of the power gain with the frequency of measurement (see Figure 6). The symbol for common emitter power gain is  $G_{pe}$ .

The parameters are voltage and current dependent, and operating point must be considered in all cases. For example, the high-frequency  $h_{fe}$  measurement at one collector voltage and current must not be used to calculate  $f_T$  directly at another voltage and/or current without considering the added effects of the different operating point.

The parameter  $f_{\alpha e}$  for present high frequency transistors usually lies in the region between 100 and 500 MHz. The term  $h_{fe}$ , measured at any frequency above this region is assumed on the 6 dB per octave portion of  $h_{fe}$  versus frequency curve and is used to calculate  $f_T$  directly.

Power gain measured at any frequency above 500 MHz is assumed on the 6 dB per octave portion of the power gain versus frequency curve and is used to calculate  $f_{max}$  directly.



**Figure 8. This Nomogram Represents  $f_T$ ,  $f_{\alpha b}$ , and Either  $h_{fbo}$  or  $h_{fe0}$**

## INSTRUCTIONS FOR CURVES AND NOMOGRAMS

The nomograms assume no shift in operating point. Known parameters used to find an unknown must be measured at the same collector voltage and collector current as the desired unknown.

Frequency scales on the nomograms are calibrated in numbers only without units. Furthermore, all nomograms

contain two frequency scales. Decimal points may be shifted on the frequency scales of any nomogram as long as they are shifted the same amount on both scales (i.e., both frequency scales of a nomogram must be multiplied by 10 to the same power). This enables the same nomogram to be used for both high and low-frequency transistors.

The nomograms assume that both power gain and current gain decrease with increasing frequency at a rate of 6 dB per octave at high frequencies.

All power gain and current gain scales (except  $h_{fbo}$  and  $h_{fe0}$ ) are calibrated in both actual magnitudes and decibel values for convenience.

### EXAMPLE 1

To find  $h_{fe0}$  when  $h_{fbo}$  is known or vice versa, enter Figure 3 with the known value and read the unknown directly. Given:  $h_{fbo} = 0.96$ . Find:  $h_{fe0}$ . Answer = 24.

### EXAMPLE 2

Figure 4 is a nomogram of  $f_T$  and  $h_{fe}$  at some frequency  $f$ , where  $f > f_{\alpha e}$ . Given:  $h_{fe}$  at 100 MHz is 6 dB. Find:  $h_{fe}$  at 75 MHz. Answer: 4, or 12 dB.

### EXAMPLE 3

There are no special instructions for the nomogram of Figure 5, merely use it to find the unknown parameter when any two are known. Given:  $h_{fe0} = 40$  and  $f_T = 400$  MHz. Find:  $f_{\alpha e}$ . Answer: 10 MHz.

### EXAMPLE 4

Figure 6 is a nomogram of  $f_{max}$  and common emitter power gain measured at some frequency  $f$  where power gain is known to be decreasing at 6 dB per octave. Given: power gain at 500 MHz is 6 dB. Find  $f_{max}$ . Answer: 800 MHz. Given:  $f_{max} = 1000$  MHz. Find: power gain at 250 MHz. Answer: 12 dB.

### EXAMPLE 5

Figure 7 is a nomogram of  $f_{\alpha b}$ ,  $f_{\alpha e}$ , and either  $h_{fbo}$  or  $h_{fe0}$ . To account for variations in this relationship with different transistor types, there are two  $f_{\alpha e}$  scales, one for  $K_\theta = 0.8$  and one for  $K_\theta = 0.9$ . Given:  $f_{\alpha e} = 1$  MHz and  $h_{fbo} = 0.90$ . Find:  $f_{\alpha b}$ . Answer: 80 kHz (assuming  $K_\theta = 0.8$ ).

### EXAMPLE 6


Figure 8 is a nomogram of  $f_T$ ,  $f_{\alpha b}$ , and either  $h_{fbo}$  or  $h_{fe0}$ . To account for variations in this relationship with different transistor types, there are two  $f_T$  scales, one for  $K_\theta = 0.8$  and one for  $K_\theta = 0.9$ . Given:  $f_T = 400$  MHz and  $h_{fbo} = 0.90$ . Find:  $f_{\alpha b}$ . Answer: 555 MHz (assuming  $K_\theta = 0.8$ ).

## REFERENCE

1. A. B. Phillips, "Transistor Engineering", McGraw-Hill Book Company, Inc., New York, N.Y., Chapter 14.



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