Phase Noise in Feedback Oscillators and Frequency Synthesizers

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Introduction:

DragonWave manufactures radio front ends for LMDS and wireless ADSL applications. For these applications, phase noise in the oscillators used for frequency translation can severely impact the quality of service, through bit-error rate degradation in the demodulators.

This white paper first discusses the phase-noise spectrum in an active device, and in a feedback oscillator. It then describes how the oscillator spectrum is modified by a phase locked (synthesizer) loop.

Device Phase Noise:

Phase-noise in an active device is generated by thermal mechanisms (white additive noise) as well as by parameter variations (shot noise).

For white additive noise, the power spectral density is flat with frequency. For a device having a noise figure of F, the power spectral density of the phase-noise is given by:

$$S_{\Delta\theta}(\omega) = 2\,FKT / P_s \qquad 1.$$

where K is Boltzman's constant,
T is absolute temperature, and
P_s is the signal level at the active device input.

For shot noise, the power spectral density of the phase-noise representation varies inversely with frequency, and is given by:

$$S_{\Delta\theta}(\omega) = \alpha / \omega \qquad 2.$$

where α is some constant.

The total power spectral density of the input phase errors can then be written as:

$$S_{\Delta\theta}(\omega) = [\alpha / \omega] + 2\,FKT / P_s. \qquad 3.$$

Oscillator Phase Noise:

The phase-noise spectrum of an oscillator is defined by the signal level, the expected noise level at the input to the active device, and the resonator characteristics.

For small phase deviations at frequency offsets less than the resonator half bandwidth, ω_o / 2Q, a phase error at the input to the active element of the oscillator results in a frequency error. This frequency error is determined by the phase-frequency relationship of the feedback network:

$$d\phi/dt = [\omega_o / 2Q] \, \Delta\theta \qquad 4.$$

Thus, for modulation rates less than the half-bandwidth of the feedback loop, the spectrum of the frequency error is identical (to within a scale factor) to the spectrum of the oscillator input phase noise, $S_{\Delta\theta}(\omega)$.

This results in the spectrum of the phase error (the integral of the frequency error) being given as:

$$S_\phi(\omega) = [\omega_o / 2Q]^2 \, \omega^{-2} \, S_{\Delta\theta}(\omega) \qquad 5.$$

for frequency offsets less than ω_o / 2Q.

For noise modulation rates large compared to this feedback bandwidth, the series feedback is not effective, and the power spectral density of the output phase, $S_\phi(\omega)$ is identical to the spectrum of the oscillator input phase noise, $S_{\Delta\theta}(\omega)$.

$$S_\phi(\omega) = S_{\Delta\theta}(\omega). \qquad 6.$$

A composite expression for the power spectral density of the output phase is:

$$S_\phi(\omega) = S_{\Delta\theta}(\omega) \, \{1 + [\omega_o / 2Q\omega]^2\} \qquad 7.$$

Combining equation 3 for the device phase noise with equation 7 for the oscillator phase noise, we find that $S_\phi(\omega)$ decreases with ω:

at 9 dB / octave up to the frequency at which the 1/f effects no longer dominate.
at 6 dB / octave from that frequency up to the feedback loop half-bandwidth.
at 0 dB / octave above that frequency up to a limit imposed by subsequent filtering.

Note that for high Q oscillators, the 1 / f effects in $S_{\Delta\theta}(\omega)$ can predominate out to an offset frequency exceeding ω_o / 2Q. Here there is no 6 dB / octave region in $S_{\Delta\theta}(\omega)$.

Also note that the portion of the phase noise spectrum $S_\phi(\omega)$ which is proportional to 1 / ω^2, leading to a 1 / ω variation for rms phase deviation, is often confused with true 1 / f effects associated with parameter variations.

Phase Locked Phase Noise:

The phase noise of an oscillator that is locked to a crystal reference by means of a phase-locked loop, is modified from the above power spectral density. Within the loop bandwidth, the phase noise of the oscillator will tend to cancel itself, leaving a phase noise essentially equal to the frequency multiplied phase noise of the crystal reference. Outside the loop bandwidth, the phase noise of the oscillator is not canceled, and will continue to decrease as indicated above, until reaching its half bandwidth, $\omega_o / 2Q$.

Since the Q of the crystal reference is very large, its half bandwidth is very small, and its frequency multiplied phase noise will remain relatively flat down to very small frequency offsets. Further, at some moderate frequency offset, this multiplied phase noise power spectral-density will be crossed by the decreasing oscillator phase noise power spectral-density.

The bandwidth of the loop should be chosen equal to the frequency offset of this crossover. The residual phase noise for the "locked" oscillator is now the multiplied phase noise of the crystal within the loop bandwidth, and the decreasing phase noise of the oscillator upside the loop bandwidth. This is shown in the Figure.

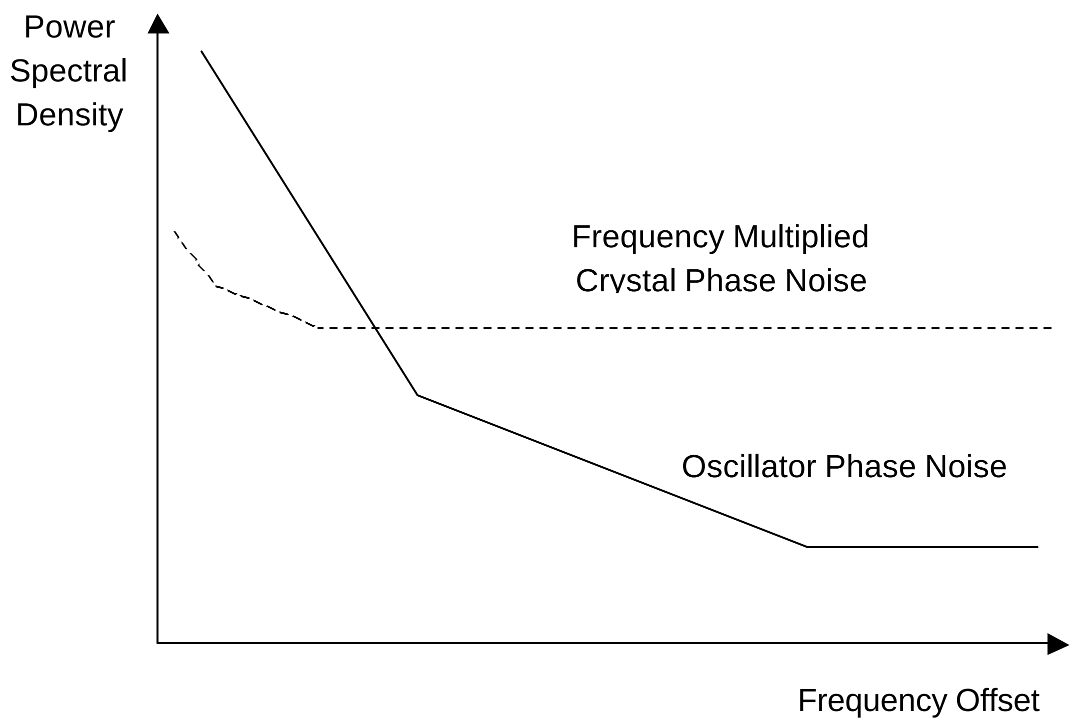

Figure 1 – Power Spectral Densities