Inequality and Social Welfare Technical Notes

TN.1. Gini index of inequality and source decomposition

To analyze the impact of various sources of income on inequality in per capita income, we use the source decomposition of the Gini index proposed by Lerman and Yitzhaki (1985; see also Garner, 1993 for an application to inequality in consumption rather than income). Denote total per capita income by y, the cumulative distribution function for total per capita income by $F(y)$ (this takes a value of zero for the poorest household and one for the richest), and the mean total per capita income across all households by $\bar{y}$. The Gini index can be decomposed as follows:

$$G_y = 2\, cov\,[y, F(y)]/\bar{y} \quad = \Sigma_i\, S_i R_i G_i$$

where G_y is the Gini index for total income, G_i is the Gini index for income y_i from source i, S_i is the share of total income obtained from source i, and R_i is the Gini correlation between income from source i and total income. The Gini correlation is defined as $R_i = cov\,[y_i, F(y)] / cov[(y_i, F(y_i)]$, where $F(y_i)$ is the cumulative distribution function of per capita income from source i. The Gini correlation R_i can take values between –1 and 1. Income from sources such as income from capital which tend to be strongly and positively correlated with total income will have large positive Gini correlations. Income from sources such as transfers tend to have smaller, and possibly negative Gini correlations. The overall (absolute) contribution of a source of income i to the inequality in total per capita income is thus $S_iR_iG_i$.

This decomposition provides a simple way to assess the impact on the inequality in total income of a marginal percentage change equal for all households in the income from a particular source. As shown in Stark, Taylor, and Yitzhaki, (1986), the impact of increasing for all households the income from source i in such a way that y_i is multiplied by $(1 + e_i)$ where e_i tends to zero, is:

$$\frac{\partial G_y}{\partial e_i} = S_i(R_i G_i - G_y)$$

This equation can be rewritten to show that the percentage change in inequality due to a marginal percentage change in the income from source i is equal to that source's contribution to the Gini minus its contribution to total income. In other words, at the margin, what matters for evaluating the redistributive impact of income sources is not their Gini, but rather the product R_iG_i which is called the pseudo Gini. Alternatively, denoting by $\eta_i = R_iG_i/G_y$ the so-called Gini income elasticity (GIE) for source i, the marginal impact of a percentage change in income from source i identical for all households on the Gini for total income in percentage terms is:

$$\frac{\partial G_y / \partial e_i}{G_y} = \frac{S_i R_i G_i}{G_y} - S_i = S_i(\eta_i - 1)$$

Thus a percentage increase in the income from a source with a GIE η_i smaller (larger) than one will decrease (increase) the inequality in per capita income. The lower the GIE is, the larger the redistributive impact will be. The GIE of income source i can be written as:

$$\eta_i = \frac{\text{cov}(x_i, F(y))}{\text{cov}(y, F(y))} * \frac{1}{S_i},$$

where x_i.is income source (or expenditure item) i per capita, y is income per capita, and S_i is the share of source i in income. The ratio of the covariances is an instrumental variable estimator of the slope of the Engel curve of source i with respect to income y, with F(y) being the instrument. Hence the ratio of the covariances can be interpreted as the slope (or the marginal propensity) of the Engel of X with respect to Y. S_i is the average propensity so that the ratio of the two yield the income elasticity of the Engel curve. Note, that at the same time, the GIE is the income elasticity of the Gini with respect to an increase in income source i.

The same decomposition can be applied to per capita consumption and its sources. The same decomposition can also be applied to the extended Gini which uses a parameter ν to emphasize various parts of the distribution. The higher the weight is, the more emphasis will be placed on the bottom part of the distribution (ν=2 for the standard Gini index):

$$G_y(\nu) = \frac{-\nu \,\text{cov}(y, [1-F]^{\nu-1})}{\bar{y}}$$

TN.2. Decomposition of the GIE into targeting and allocation GIEs

A decomposition of the GIE proposed by Wodon and Yitzhaki (2001) can be used to differentiate between two properties of a program that can affect its impact on inequality: targeting and the allocation mechanism among participants (internal progressivity). The decomposition enables the analyst to assess whether the (lack of) performance of social programs and policies is due to either the selection mechanism for participants or the allocation of benefits among program participants. To differentiate between targeting and internal progressivity, define z as the targeting instrument:

$$z = \begin{pmatrix} \bar{x}_p & if & h \in P \\ 0 & if & h \notin P \end{pmatrix}$$

That is, z is equal to the mean benefit among participants for households who participate in the program and it is zero for households who do not participate (one could substitute the average benefit by an indicator which is equal to one without affecting the results.) The variable z is an indicator of targeting because it is only concerned with whom is affected by the program, rather than with the actual benefit received. Using this definition of z, we can rewrite the GIE as a product of two elasticities as follows:

$$\eta = \left(\frac{\text{cov}(z, F(y))}{\text{cov}(y, F(y))} \frac{\bar{y}}{\bar{z}}\right)\left(\frac{\text{cov}(x, F(y))}{\text{cov}(z, F(y))} \frac{\bar{z}}{\bar{x}}\right) = \eta_T \eta_A$$

The first term is the progressivity among participants (allocation effect). The second term is related to the targeting of the program (targeting effect). The distributional impact of a program depends on the product of its targeting and allocation elasticities. Good targeting, for example, can be offset by a bad allocation mechanism among program beneficiaries. This equation is useful to assess whether the (lack of) performance of a program is due to its targeting or to the allocation of benefits among beneficiaries. But one can go further by establishing bounds for the values of the targeting and allocation elasticities. Specifically, the minimum and maximum values of the targeting elasticity depend on the share the population participating in the program and the overall Gini. Denoting by p the share of the population participating in the program, and by G_y the overall Gini, it is shown in the appendix that :

$$-\frac{(1-p)}{G_y} \leq \eta_T \leq \frac{(1-p)}{G_y}.$$

The lower bound increases with the proportion of the population reached by the program and the level of income inequality. The relationship between the lower bound and the share of program participants is straightforward. The more households the program reaches, the less effective targeting can be because each additional participating household makes it more difficult to focus resources on the poorest. If all households participate, p=1 and the lower bound is zero. The fact that the targeting capacity declines with the overall level of inequality is perhaps more surprising, because one might expect that the higher the inequality, the higher the potential for redistribution through targeting. The intuitive explanation is that the higher the inequality, the further apart households are from each other, so that adding a small amount of resources to the program participants does not reduce inequality by a lot (remember that the elasticities capture the impact on inequality and social welfare on e per dollar basis). A similar reasoning applies for the intuition regarding the upper bound, and the two bounds are symmetric around zero.

Lower and upper bounds can also be provided for the allocation elasticity. As shown in the appendix, the minimum and maximum values of the allocation elasticity depend on the share the population participating in the program, but not on the overall Gini:

$$-\frac{1}{(1-p)} \leq \eta_A \leq \frac{1}{(1-p)}.$$

As the share of the population participating in the program increases, the interval for the allocation elasticity increases as well, because a higher participation rates provides more opportunities for differenciation in the allocation of the benefits among participants of the program. It is important to note that the interpretation of what a good elasticity need not be the same for the targeting and allocation elasticities. In the case of the targeting elasticity, one would hope to obtain an elasticity below zero, which would indicate a good targeting performance. But if the targeting elasticity is below zero, one would hope to have an allocation elasticity above zero in order to keep the overall elasticity negative. If the targeting elasticity is positive, suggesting bad targeting, one would hope to have a negative allocation elasticity. It should also be emphasized that the interpretation of the upper and lower bounds for both elasticities changes depending on whether we are dealing with taxes or transfers. In the case of targeting for example, when comparing transfers, the lower bound is typically the best that can be achieved, while when dealing with taxes, it is the upper bound that one would like to reach. Note finally that the bounds for the targeting and allocation elasticities enable us in principle to compare the targeting and allocation effectiveness of programs with different participation rate, since the bounds depend on that participation rate. (The role played by the overall per capita income Gini in the bounds for the targeting elasticity is less important since the Gini is identical for all programs at any given point in time.)

TN.3. Social Welfare function, growth, and redistribution

To assess the impact on welfare of government programs per dollar spent in each program, following Yitzhaki (2000), we denote by $\bar{y}$ the mean income in the population and by G_y the Gini index of income inequality. A common welfare function used in the literature is $W=\bar{y}(1–G_y)$. The higher the mean income, the higher the level of social welfare, but the higher the inequality, the lower the aggregate level of welfare. This welfare function takes into account not only absolute, but also relative deprivation (people assess their own level of welfare in part by comparing themselves with others). Using the implicit distributional weights embodied in this welfare function, we can derive the marginal gains from additional investments in government programs. If $\bar{x}$ denotes the mean benefit of a social program x across the whole population, and if η is the Gini income elasticity of that program (defined below), increasing at the margin the funds allocated to the program by multiplying the outlays by $1 + \Delta$ for all program participants, with Δ small, will result in a marginal social welfare gain equal to:

$$\Delta W = (\Delta \bar{x})(1 - \eta\, G_y)$$

This equation makes it clear that considerations related to both growth (as represented by the mean marginal benefit $\Delta \bar{x}$) and distribution (as represented by the Gini income elasticity η times the Gini index G) must be taken into account in program evaluations.