HYBRID ARQ IN WIRELESS NETWORKS

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AUTOMATIC REPEAT REQUEST

- The receiving end detects frame errors and requests retransmissions.
- P_e is the frame error rate, the average number of transmissions is

$$1 \cdot (1 - P_e) + \cdots + n \cdot P_e^{n-1}(1 - P_e) + \cdots = \frac{1}{1 - P_e}$$

- Hybrid ARQ uses a code that can correct some frame errors.
- In HARQ schemes
  - the average number of transmissions is reduced, but
  - each transmission carries redundant information.

ARQ

An Example

- Decoding the name of an information theorist from its noisy version:

EMRE

ARQ
An Example

- Decoding the name of an information theorist from its noisy version:

EMRE

ARQ

An Example

- Decoding the name of an information theorist from its noisy version:

IMRE

ARQ
An Example

- Decoding the name of an information theorist from its noisy version:

IMRE

- Increasing redundancy:

E M R E

ARQ

An Example

- Decoding the name of an information theorist from its noisy version:

IMRE

- Increasing redundancy:

E M R E T E L A T A R

ARQ
An Example

- Decoding the name of an information theorist from its noisy version:

IMRE

- Increasing redundancy:

E M R E T E L A T A R
I M R E

ARQ

An Example

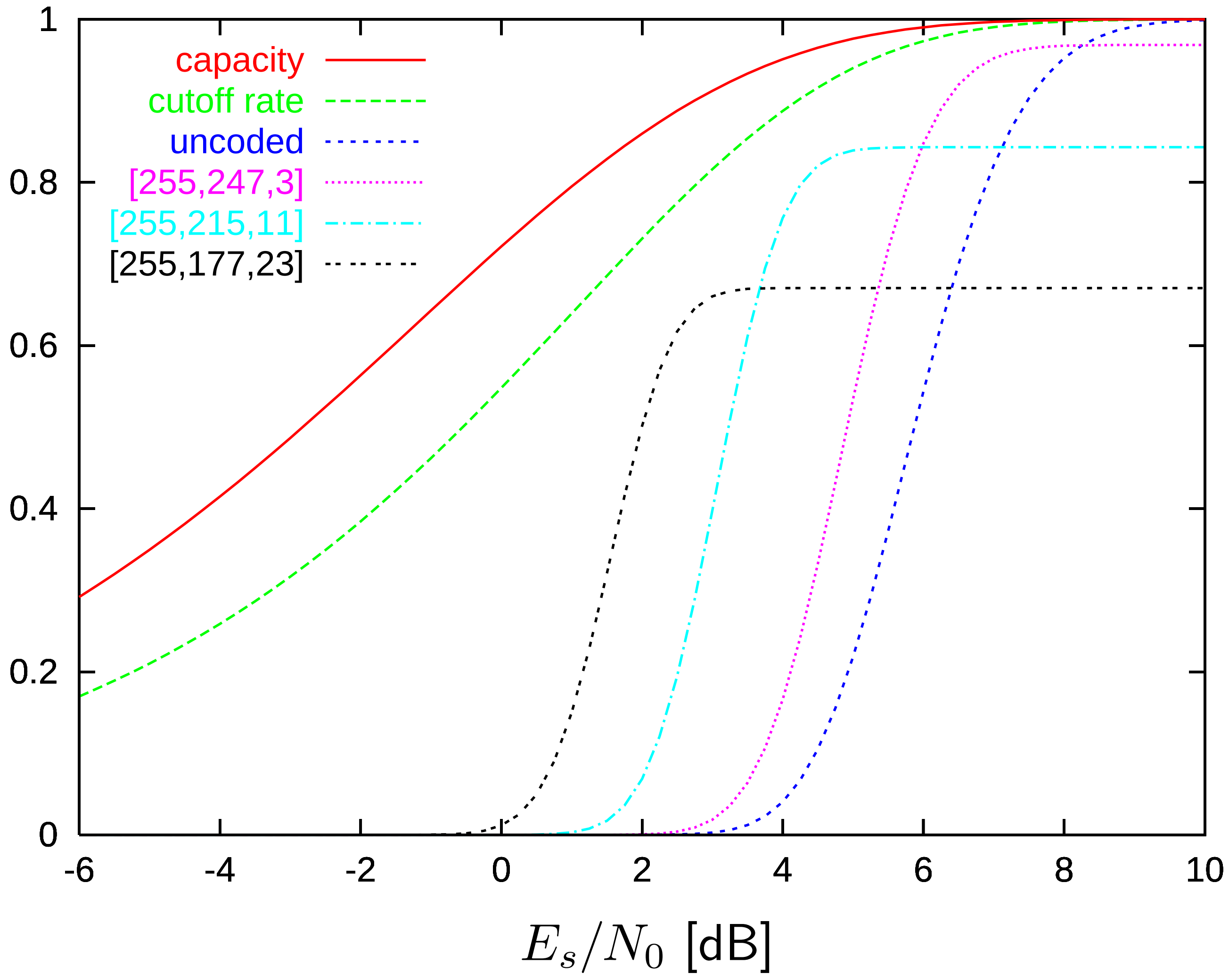
- Decoding the name of an information theorist from its noisy version:

IMRE

- Increasing redundancy:

E	M	R	E	T	E	L	A	T	A	R
I	M	R	E	C	S	I	S	Z	A	R

THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded



TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

E M R E T E L A T A R

- Rate compatible:

M R E

TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

E M R E T E L A T A R

- Rate compatible:

M R E A R

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E M R E T E L A T A R

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TYPE II HYBRID ARQ
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- Puncturing:

E M R E T E L A T A R

- Rate compatible:

E M R E T E L A T A R

- Not rate compatible:

M R E

TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

E M R E T E L A T A R

- Rate compatible:

E M R E T E L A T A R

- Not rate compatible:

M E A R

TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

E M R E T E L A T A R

- Rate compatible:

E M R E T E L A T A R

- Not rate compatible:

M E T E L A A

TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

E M R E T E L A T A R

- Rate compatible:

E M R E T E L A T A R

- Not rate compatible:

E E T E L T A

TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

E M R E   T E L A T A R

- Rate compatible:

E M R E   T E L A T A R

- Not rate compatible:

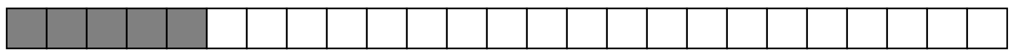
E   E   T E L   T A

TYPE II HYBRID ARQ
Incremental Redundancy

- Information bits are encoded by a (low rate) mother code.
- Information and a selected number of parity bits are transmitted.
- If a retransmission is not successful:
  - transmitter sends additional selected parity bits
  - receiver puts together the new bits and those previously received.
- Each retransmission produces a codeword of a stronger code.
- Family of codes obtained by puncturing of the mother code.

INCREMENTAL REDUNDANCY

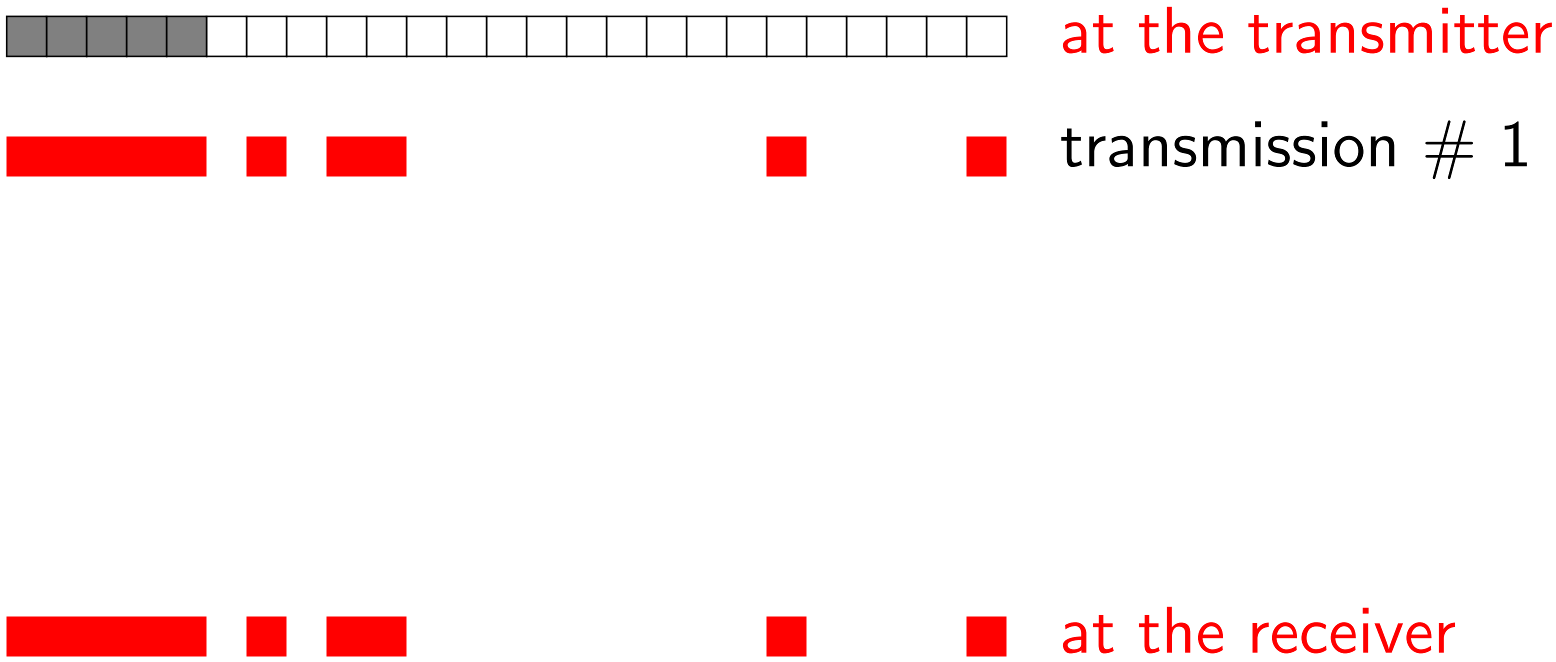
A Rate $1/5$ Mother Code



at the transmitter

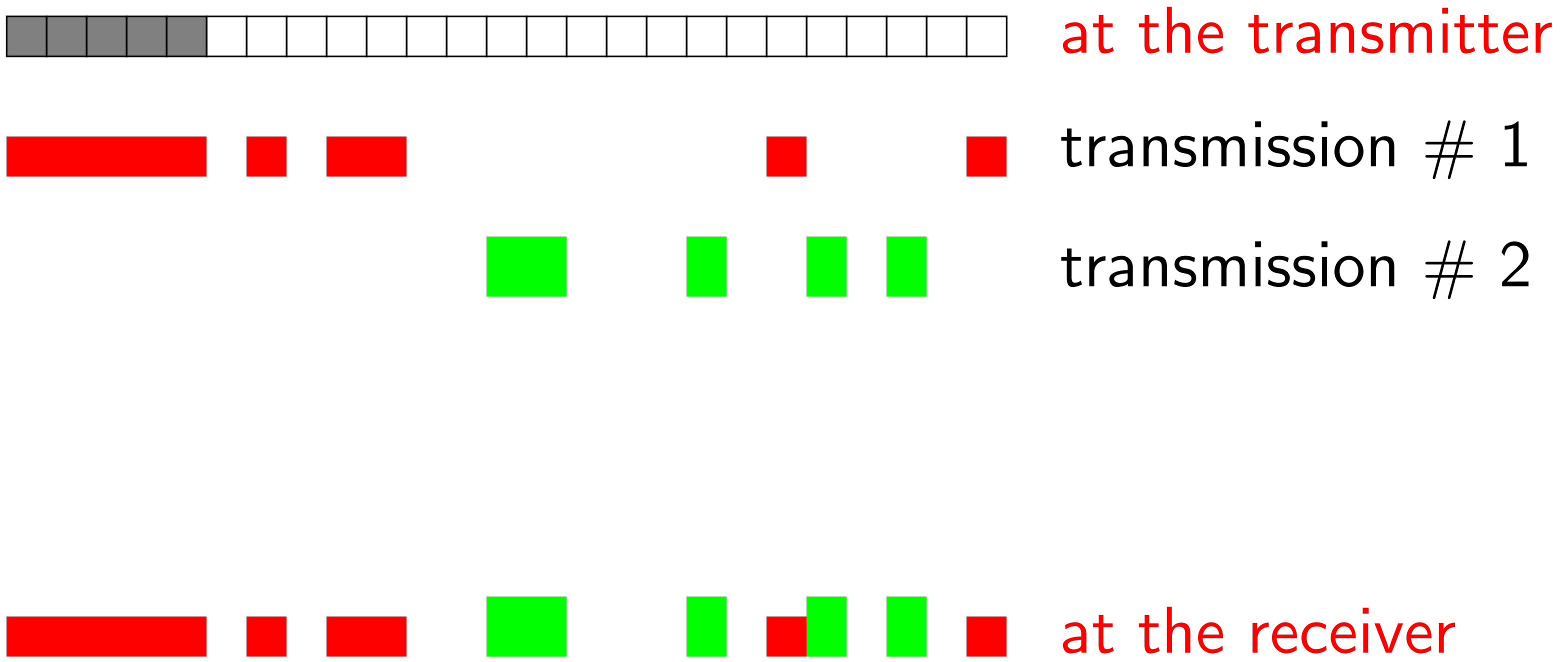
INCREMENTAL REDUNDANCY

A Rate $1/5$ Mother Code



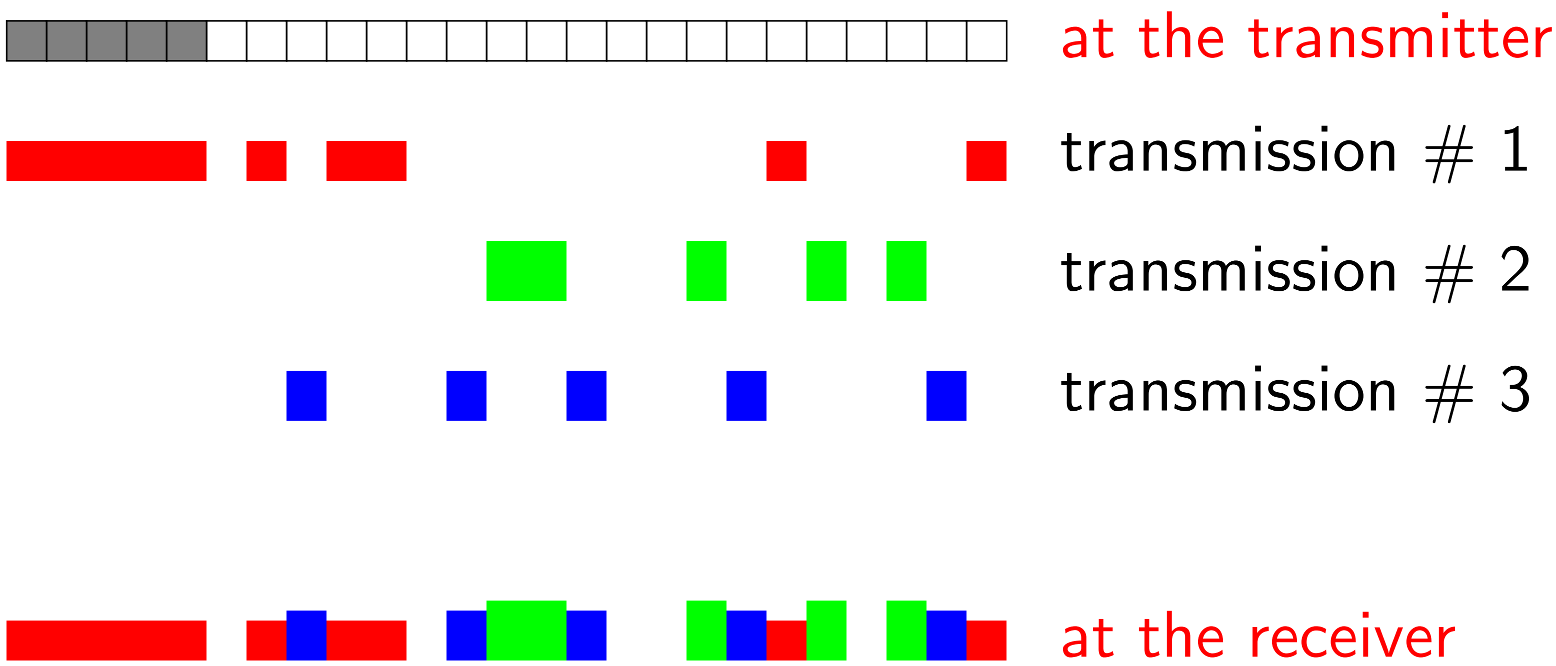
INCREMENTAL REDUNDANCY

A Rate $1/5$ Mother Code



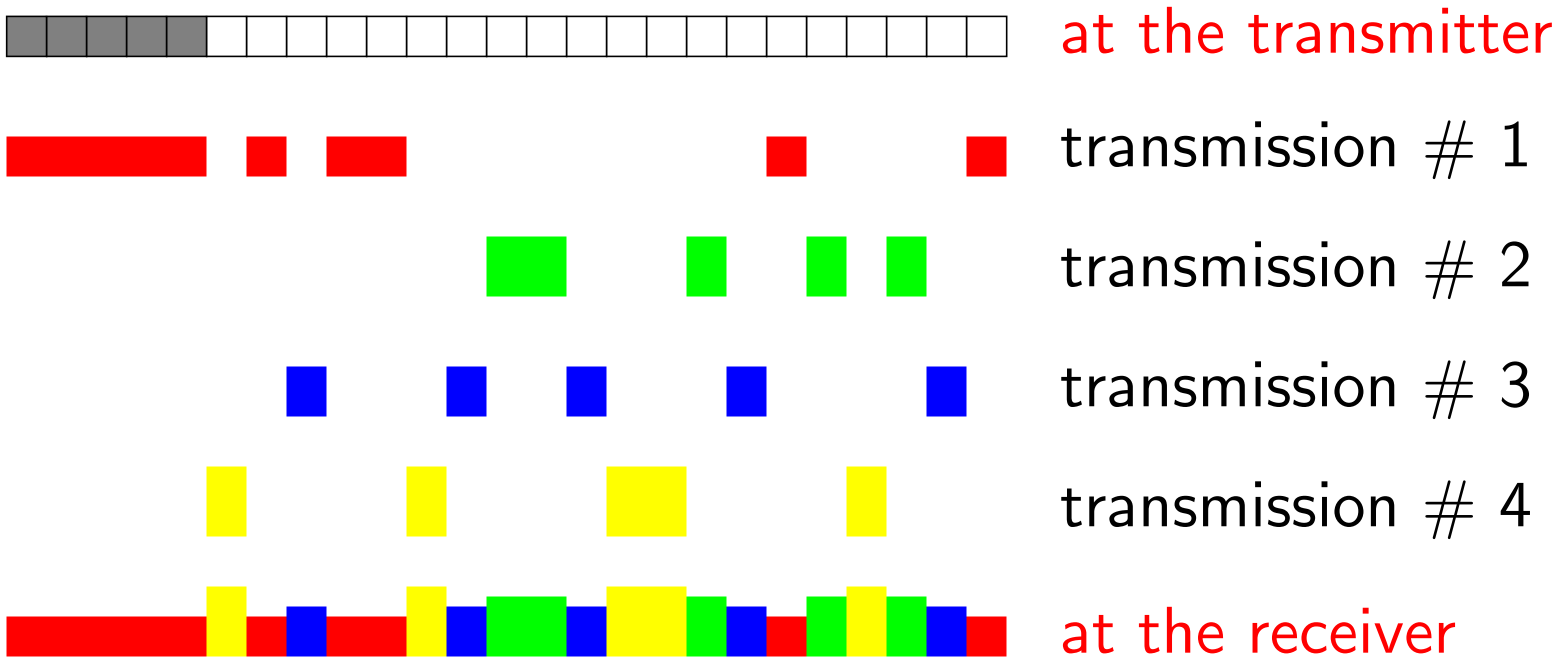
INCREMENTAL REDUNDANCY

A Rate $1/5$ Mother Code

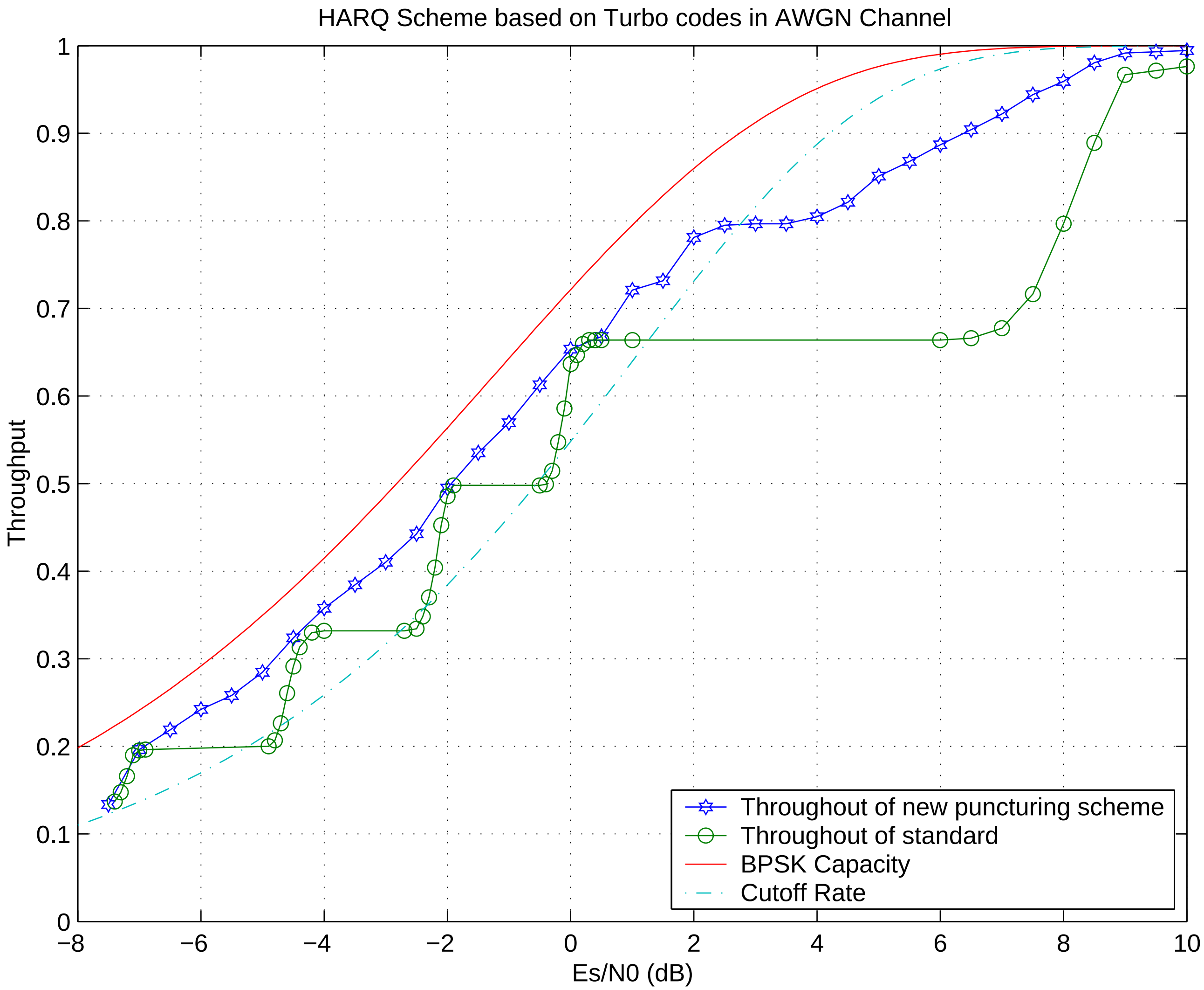


INCREMENTAL REDUNDANCY

A Rate $1/5$ Mother Code

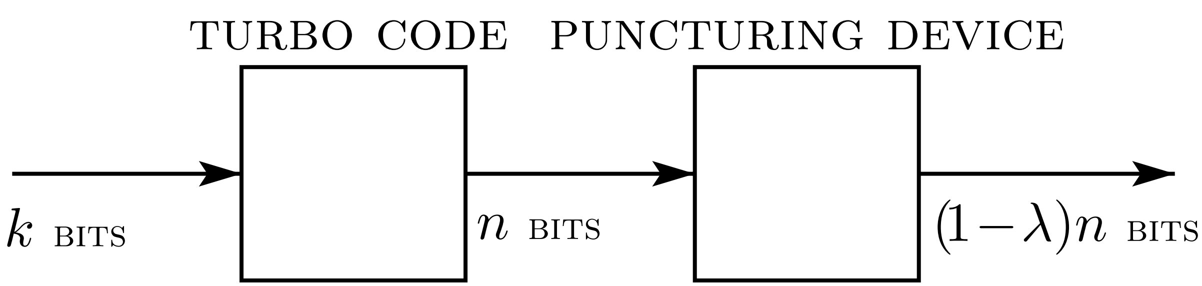


THROUGHPUT IN HYBRID ARQ



RANDOMLY PUNCTURED CODES

- The mother code is an (n, k) rate R turbo code.
- Each bit is punctured independently with probability λ.
- The expected rate of the punctured code is $R/(1-\lambda)$.
- For large n we have



A FAMILY OF RANDOMLY PUNCTURED CODES

Rate Compatible Puncturing

- The mother code is an (n, k) rate R turbo code.
- λ_j for $j = 1, 2, \ldots, m$ are puncturing rates, $\lambda_j > \lambda_k$ for $j < k$.
- If the i-th bit is punctured in the k-th code and $j < k$, then it was punctured in the j-th code.
- θ_i for $i = 1, 2, \ldots, n$ are uniformly distributed over $[0, 1]$.
- If $\theta_i < \lambda_l$, then the i-th bit is punctured in the l-th code.

MEMORYLESS CHANNEL MODEL

- Binary input alphabet $\{0, 1\}$ and output alphabet $\mathcal{Y}$.
- Constant in time with transition probabilities $W(b|0)$ and $W(b|1)$, $b \in \mathcal{Y}$.
- Time varying with transition probabilities at time i $W_i(b|0)$ and $W_i(b|1)$, $b \in \mathcal{Y}$.
- $W_i(\cdot|0)$ and $W_i(\cdot|1)$ are known at the receiver.

PERFORMANCE MEASURE
Time Invariant Channel

- Sequence $\boldsymbol{x} \in \mathcal{C} \subseteq \{0,1\}^n$ is transmitted, and $\boldsymbol{x'}$ decoded.

- Sequences $\boldsymbol{x}$ and $\boldsymbol{x'}$ are at Hamming distance d.

- The probability of error $P_e(\boldsymbol{x}, \boldsymbol{x'})$ can be bounded as

$$P_e(\boldsymbol{x}, \boldsymbol{x'}) \leq \gamma^d = \exp\{-d\alpha\},$$

where γ is the Bhattacharyya noise parameter:

$$\gamma = \sum_{b \in \mathcal{Y}} \sqrt{W(b|0)W(b|1)}$$

and $\alpha = -\log\gamma$ is the Bhattacharyya distance.

PERFORMANCE MEASURE

- An (n, k) binary linear code $\mathcal{C}$ with A_d codewords of weight d.

- The union-Bhattacharyya bound on word error probability:

$$P_W^{\mathcal{C}} \leq \sum_{d=1}^{n} A_d e^{-\alpha d}.$$

- Weight distribution A_d for a turbo code?

- Consider a set of codes $[\mathcal{C}]$ corresponding to all interleavers.

- Use the average $\overline{A}_d^{[\mathcal{C}](n)}$ instead of A_d for large n.

TURBO CODE ENSEMBLES
A Coding Theorem by Jin and McEliece

- There is an ensemble distance parameter $c_0^{[\mathcal{C}]}$ s.t. for large n,

$$\overline{A}_d^{[\mathcal{C}](n)} \leq \exp\left(dc_0^{[\mathcal{C}]}\right) \text{ for large enough } d.$$

- For a channel whose Bhattacharyya distance $\alpha > c_0^{[\mathcal{C}]}$, we have

$$\overline{P}_W^{[\mathcal{C}](n)} = O(n^{-\beta}).$$

- $c_0^{[\mathcal{C}]}$ is the ensemble noise threshold.

PUNCTUREDTURBO CODE ENSEMBLES

- Is there the punctured ensemble noise threshold $c_0^{[\mathcal{C}_P]}$:

$$\overline{A}_j^{[\mathcal{C}_P](n)} \leq \exp\left(j c_0^{[\mathcal{C}_P]}\right) \quad \text{for large enough } n \text{ and } j.$$

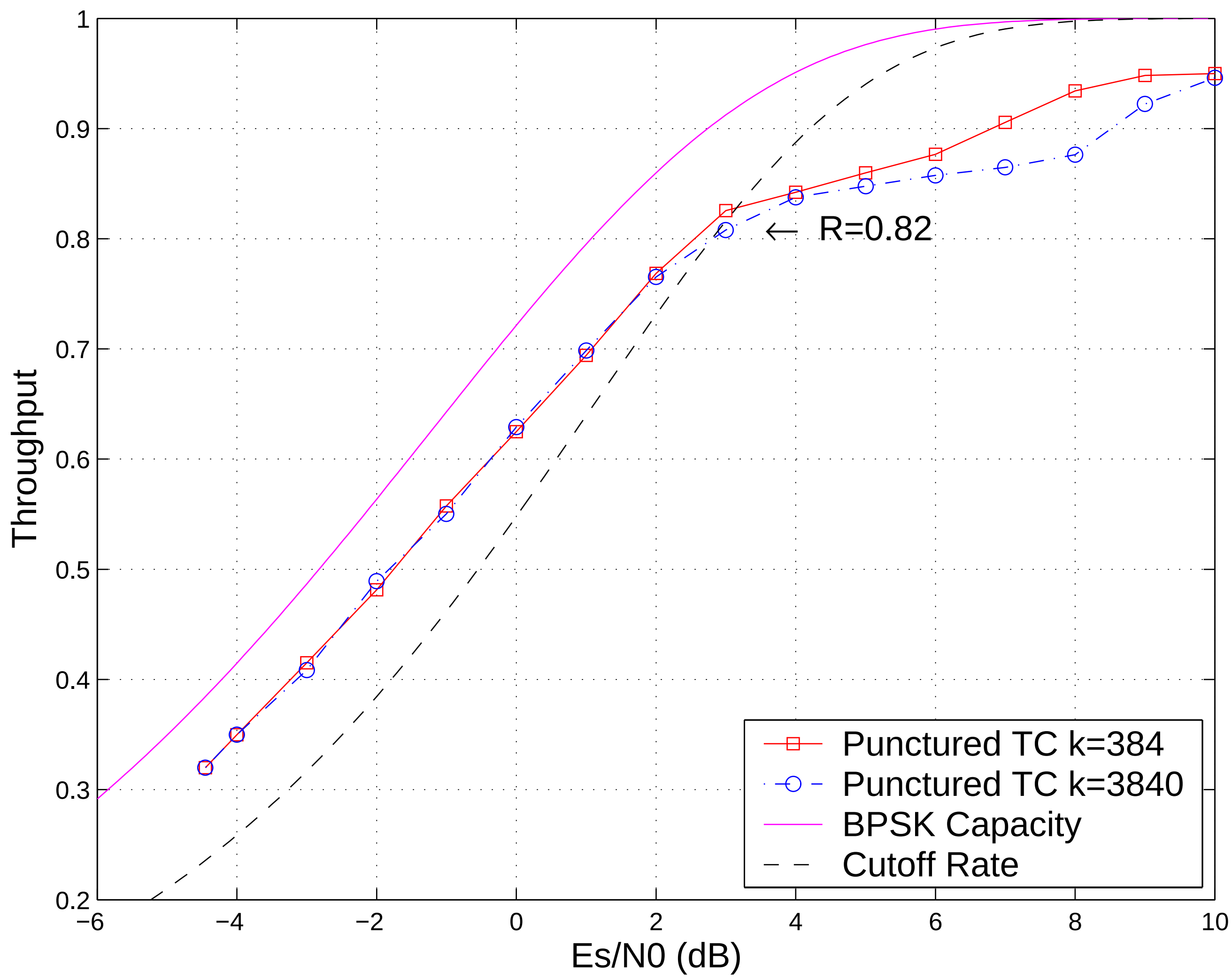
- The expected number of codewords of weight j:

$$\overline{A}_j^{[\mathcal{C}_P](n)} = \sum_{d \geq j} \overline{A}_d^{[\mathcal{C}](n)} \binom{d}{j} \lambda^{d-j} (1-\lambda)^j$$

- If $\log \lambda < -c_0^{[\mathcal{C}]}$,

$$c_0^{[\mathcal{C}_P]} \leq \log \left[\frac{1-\lambda}{\exp\left(-c_0^{[\mathcal{C}]}\right) - \lambda} \right].$$

PUNCTURED TURBO CODE ENSEMBLES



HARQ MODEL

- There are at most m transmissions.
- $\mathcal{I} = \{1, \ldots, n\}$ is the set indexing the bit positions in a codeword.
- $\mathcal{I}$ is partitioned in m subsets $\mathcal{I}(j)$, for $1 \leq j \leq m$.
- Bits at positions in $\mathcal{I}(j)$ are transmitted during j-th transmission.
- The channel remains constant during a single transmission:

$$\gamma_i = \gamma(j) \quad \text{for all } i \in \mathcal{I}(j).$$

PERFORMANCE MEASURE
Time Varying Channel

- Let $W^n(\boldsymbol{y}|\boldsymbol{x}) = \prod_{i=1}^{n} W_i(y_i|x_i)$.
- Sequence $\boldsymbol{x} \in \mathcal{C} \subseteq \{0,1\}^n$ is transmitted, and $\boldsymbol{x'}$ decoded.
- The probability of error $P_e(\boldsymbol{x}, \boldsymbol{x'})$ can be bounded as

$$P_e(\boldsymbol{x}, \boldsymbol{x'}) \leq \sum_{\boldsymbol{y} \in \mathcal{Y}^n} \sqrt{W^n(\boldsymbol{y}|\boldsymbol{x}) W^n(\boldsymbol{y}|\boldsymbol{x'})}$$

$$= \prod_{i=1}^{n} \Big(\sum_{b \in \mathcal{Y}} \sqrt{W_i(b|x_i) W_i(b|x'_i)} \Big)$$

$$\leq \prod_{i: x_i \neq x'_i} \gamma_i$$

HARQ PERFORMANCE

- d_j is the Hamming distance between $\boldsymbol{x}$ and $\boldsymbol{x'}$ over $\mathcal{I}(j)$.
- The probability of error $P_e(\boldsymbol{x}, \boldsymbol{x'})$ can be bounded as

$$P_e(\boldsymbol{x}, \boldsymbol{x'}) \leq \prod_{j=1}^{m} \gamma(j)^{d_j}$$

- $A_{d_1 \ldots d_m}$ is the number of codewords with weight d_j over $\mathcal{I}(j)$.
- The union bound on the ML decoder word error probability:

$$P \leq \sum_{d_1=1}^{|\mathcal{I}(1)|} \cdots \sum_{d_m=1}^{|\mathcal{I}(m)|} A_{d_1 \ldots d_m} \prod_{j=1}^{m} \gamma(j)^{d_j}$$

HARQ PERFORMANCE

Random Transmission Assignment

- A bit is assigned to transmission j with probability α_j.
- d is the weight of the original codeword.
- d_j is the weight of the d-th transmission sub-word.
- The probability that the sub-word weights are $d_1, d_2 \ldots, d_m$ is

$$\binom{d}{d_1}\binom{d-d_1}{d_2}\cdots\binom{d-d_1\cdots-d_{m-1}}{d_m}\alpha_1^{d_1}\alpha_2^{d_2}\cdots\alpha_m^{d_m}$$

HARQ PERFORMANCE

Random Transmission Assignment

- The union bound on the ML decoder word error probability:

$$P \le \sum_{d_1=1}^{|\mathcal{I}(1)|} \cdots \sum_{d_m=1}^{|\mathcal{I}(m)|} A_{d_1 \ldots d_m} \prod_{j=1}^{m} \gamma(j)^{d_j}$$

- The expected value of the union bound is

$$\sum_{d} A_d \Big(\sum_{j=1}^{m} \gamma(j) \alpha_j \Big)^d .$$

- The average Bhattacharyya noise parameter:

$$\overline{\gamma} = \sum_{j=1}^{m} \gamma(j) \alpha_j$$

A RANDOMLY PUNCTURED TURBO CODE
An Example of Random Transmission Assignment

- The puncturing probability is λ.

- Transmission over the channel with noise parameter γ.

- Equivalent to having two transmissions:
  - first with assignment probability $(1-\lambda)$ and noise parameter γ;
  - second with assignment probability λ and noise parameter 1.

- The average noise parameter is $\overline{\gamma} = (1-\lambda)\gamma + \lambda$.

- The requirement $-\log \overline{\gamma} > c_0^{[\mathcal{C}]}$ translates into

$$-\log \gamma > \log \left[\frac{1-\lambda}{\exp\left(-c_0^{[\mathcal{C}]}\right) - \lambda} \right].$$

INCREMENTAL REDUNDANCY

Concluding Remarks