Mobile Radio Communications

Session 4: Modulation & transmission

Why modulation ?

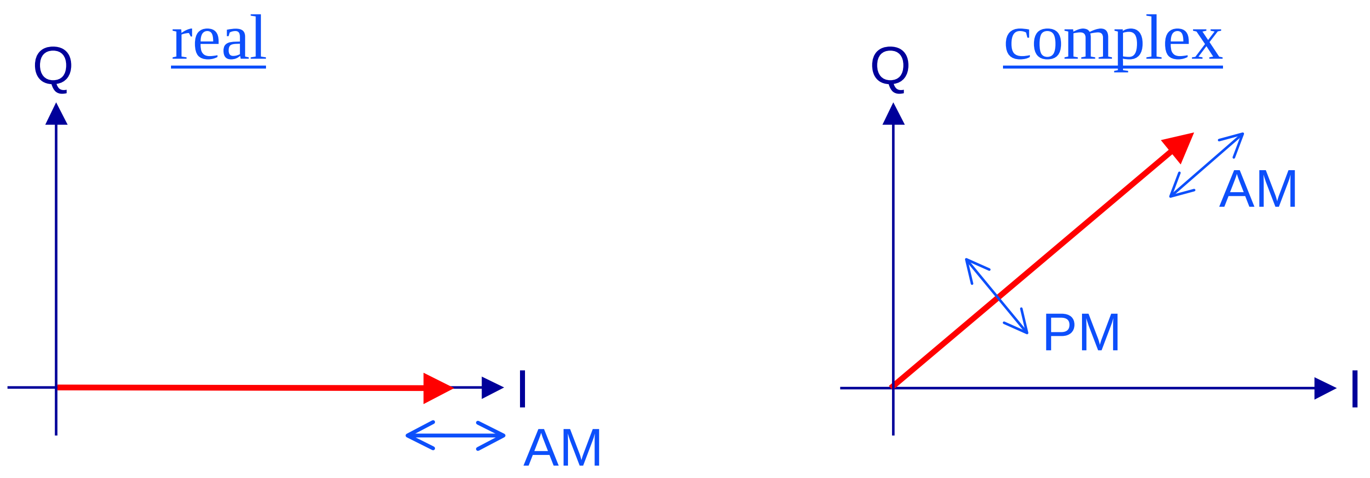
- RF carrier as transmission medium
- spectral efficiency
- robustness on radio path
- analog versus digital information

Carrier modulation

- **amplitude/phase**
- **phasor / complex baseband representation**
- **I and Q modulation:**

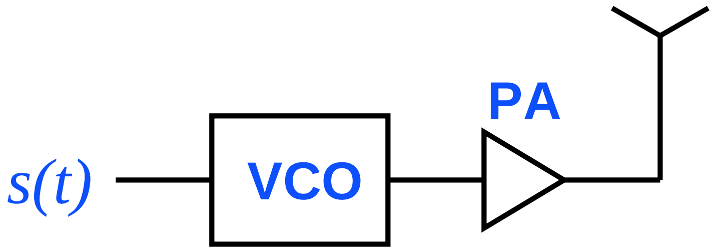
  cos and sin

  in-phase and quadrature component



Constant-envelope modulation

- RF amplitude constant
- class C amplification (efficiency)
- limiting receiver
- non-linear TX-RX operations
- information in phase only (but be careful …)



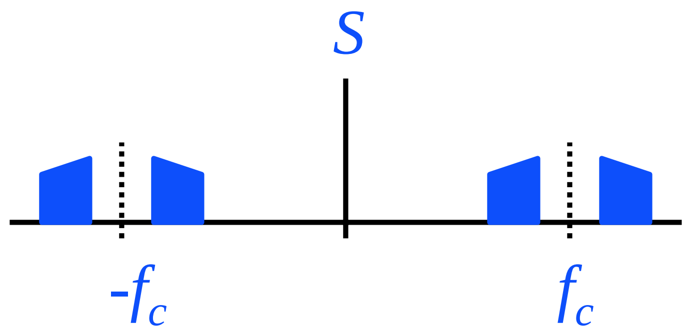
Analog modulation

- **Information is analog**
- **AM, PM, and FM**

AM

$$s_{AM}(t) = A_c[1 + m(t)]\cos 2\pi f_c t$$

$$k = \frac{A_m}{A_c} \quad if \quad m(t) = \frac{A_m}{A_c}\cos 2\pi f_c t$$



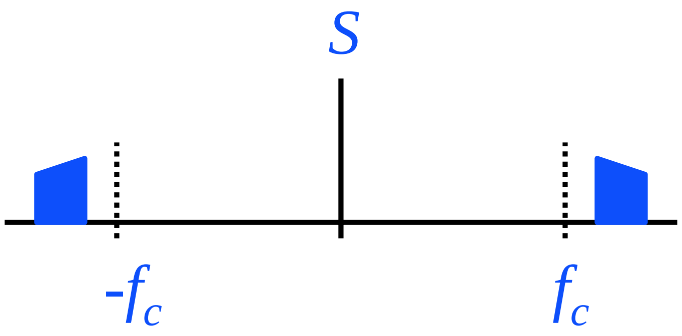
SSB modulation

- **AM: redundant information (two side bands)**
- **Remove one side band → <u>Single Side Band</u>**
- **Complex !**

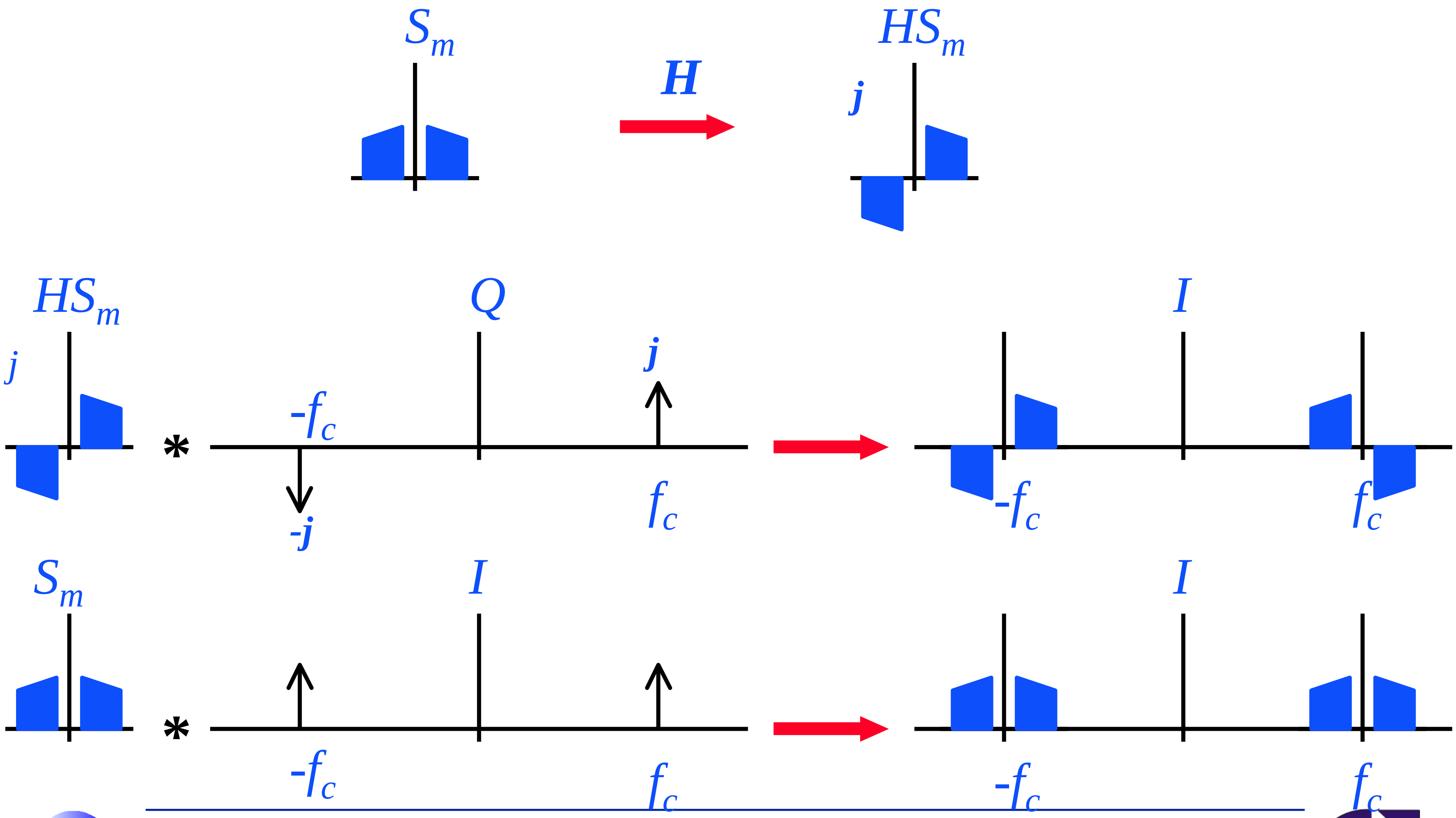
<u>SSB</u>

$$s_{SSB}(t) = A_c\left(m(t)\cos 2\pi f_c t \mp \hat{m}(t)\sin 2\pi f_c t\right)$$

$\hat{m}(t)$ Hilbert transform

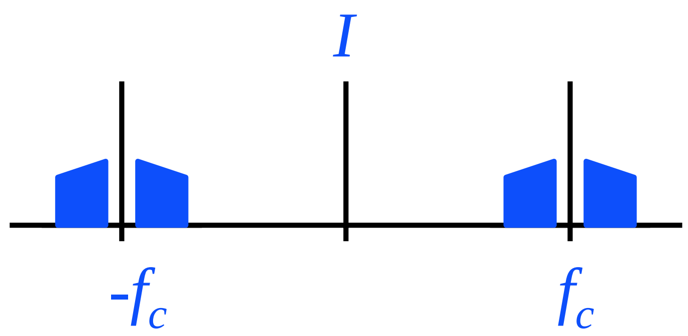


SSB generation

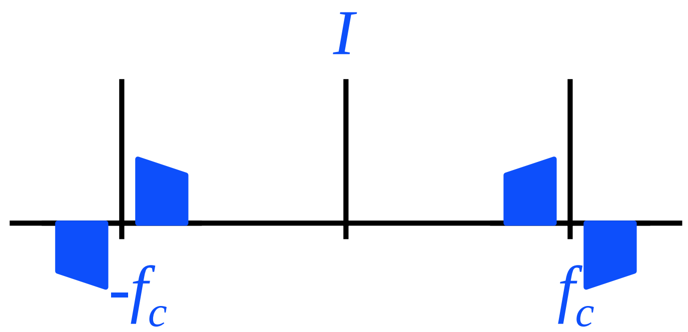


SSB generation

$$A_c m(t)\cos 2\pi f_c t$$



$$A_c \hat{m}(t)\sin 2\pi f_c t$$



- Addition: lower side bands
- Subtraction: upper side bands

SSB features

- Amplitude and phase modulation
- Bandwidth efficient but poor performance in radio channel
- Needs accurate frequency tuning
- Pilot tone required

Angle modulation

- **Frequency or phase**
- **Constant envelope**

FM

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

k_f frequency deviation in V/Hz

$$\beta_f = \frac{k_f A_m}{W} = \frac{\Delta f}{W}$$

$$if \quad m(t) = A_m \cos 2\pi f_m t$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

Angle modulation

PM

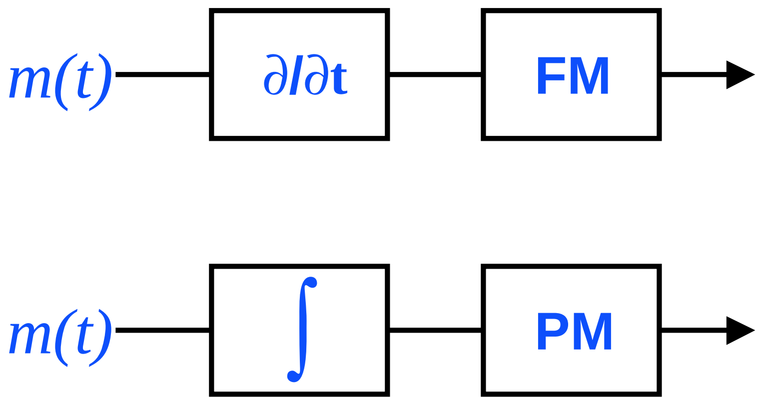
$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + k_\theta m(t)\right]$$

k_θ phase deviation in rad/Hz

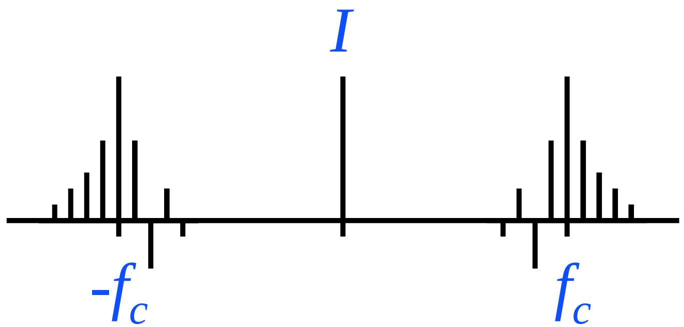
$$\beta_p = k_\theta A_m = \Delta\theta$$

PM

FM



FM modulation

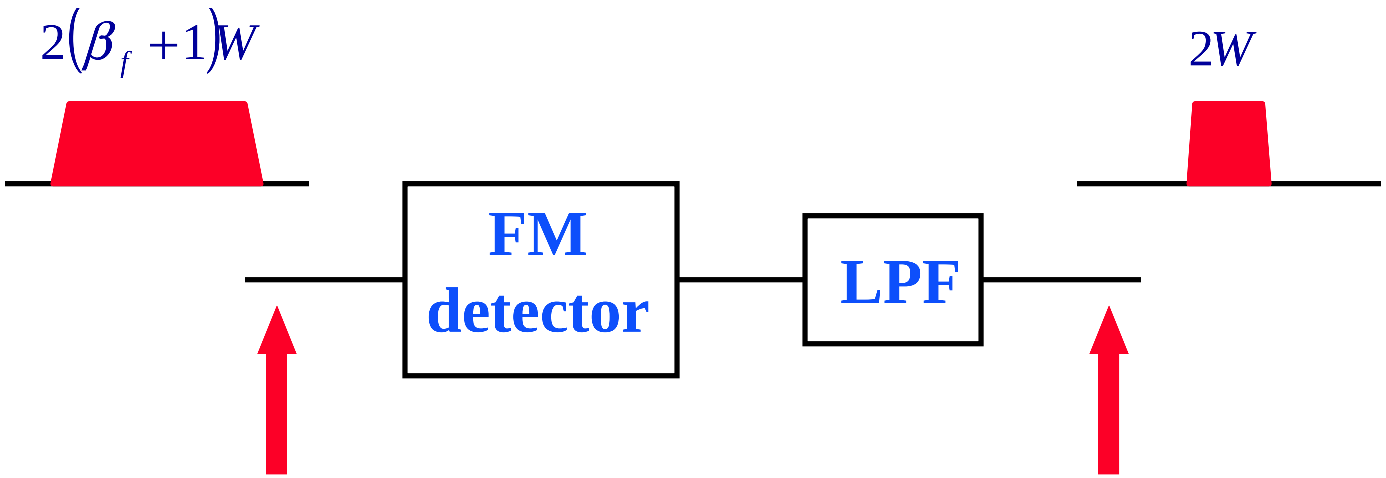


Carson's rule:

$$B_T = 2(\beta_f + 1) f_m \qquad \beta_f < 1$$

$$B_T = 2\Delta f \qquad \beta_f >> 1$$

FM SNR performance



$$SNR_{in} = \frac{A_c^2 / 2}{2 N_0 (\beta_f + 1) W}$$

$$SNR_{out} = 6(\beta_f + 1)\beta_f^2 \overline{\left(\frac{m(t)}{V_p}\right)^2} SNR_{in}$$

FM SNR performance

$$SNR_{out} = 3\beta_f^2 \frac{P_{carrier}}{N_0 W}$$

$$P_{carrier} = \frac{A_c^2}{2}$$

Digital modulation

- **Information is digital**
- **binary modulation:** bit $b_i \rightarrow$ symbol s_k
- ***m*-ary modulation:** $\{b_i, b_{i+1} \ldots b_{i+N-1}\} \rightarrow s_k$

  $\log_2 m$ bits/symbol

  $m \rightarrow \eta_b \quad (\eta_b = R_b/W)$

- E_b **energy per bit:** $P_{carrier} \times T_b$
- E_s **energy per symbol:** $P_{carrier} \times T_s$
- N_0 **noise spectral density:** P_{noise}/W

Digital modulation

- **symbol duration T_s;** **symbol rate $R_s = 1/T_s$**

  symbol rate = baud rate

- **bit duration T_b;** **bit rate $R_b = 1/T_b$**

$$R_b = R_s \log_2 m$$

Shannon's channel capacity

$$C = W \log_2(1 + SNR)$$

$$SNR = \frac{P_s}{P_N} = \frac{E_b / T_b}{N_0 \cdot W} = \frac{E_b}{N_0} \cdot \frac{R_b}{W}$$

Maximum capacity: $C = R_b$

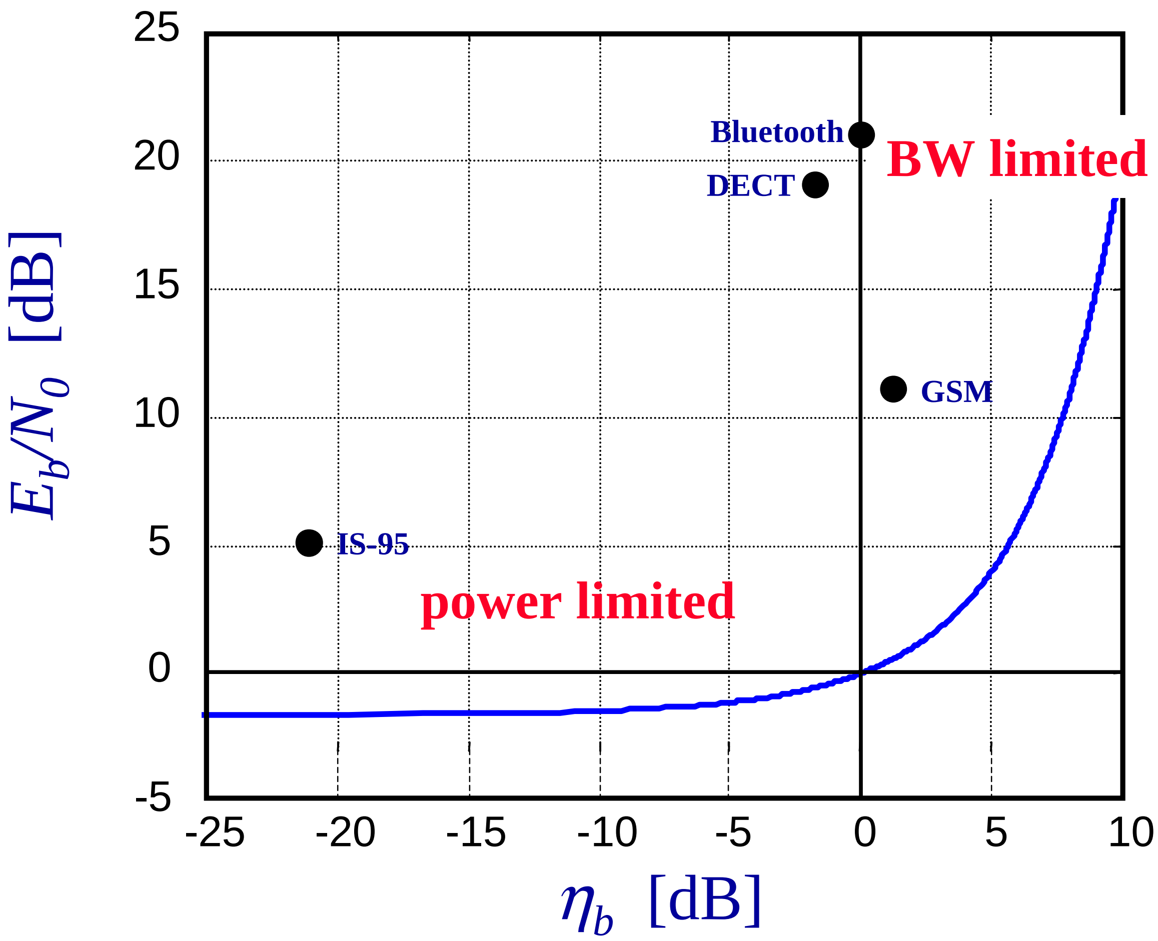
Shannon's channel capacity

$$\frac{R_b}{W} = \log_2 \left(1 + \frac{E_b}{N_0} \cdot \frac{R_b}{W}\right)$$

$$\frac{E_b}{N_0} = \frac{2^{R_b/W} - 1}{R_b/W} = \frac{2^{\eta_b} - 1}{\eta_b}$$

η_b spectral efficiency b/s/Hz

Power-bandwidth trade-off



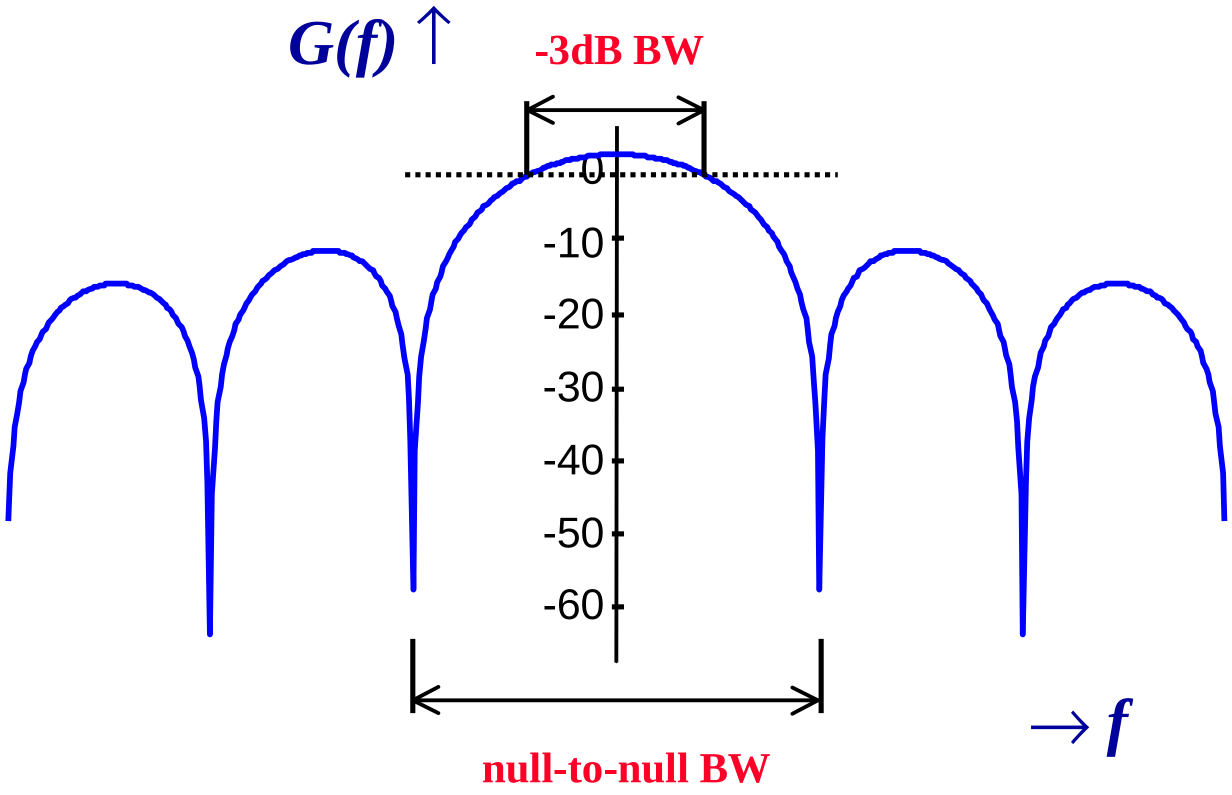
Power spectral density

Wiener-Kinchine theorem:

$$G_v(f) \overset{\boldsymbol{F}}{\longleftrightarrow} R_v(\tau)$$

autocorrelation: $R_v(\tau) = E\big(v(t)v(t+\tau)\big)$

Bandwidth definitions



absolute BW: freq. range for which $G(f) > 0$

99% BW: freq. range where 99% of power is

Digital transmission chain

Information bits 011000110001101101110010

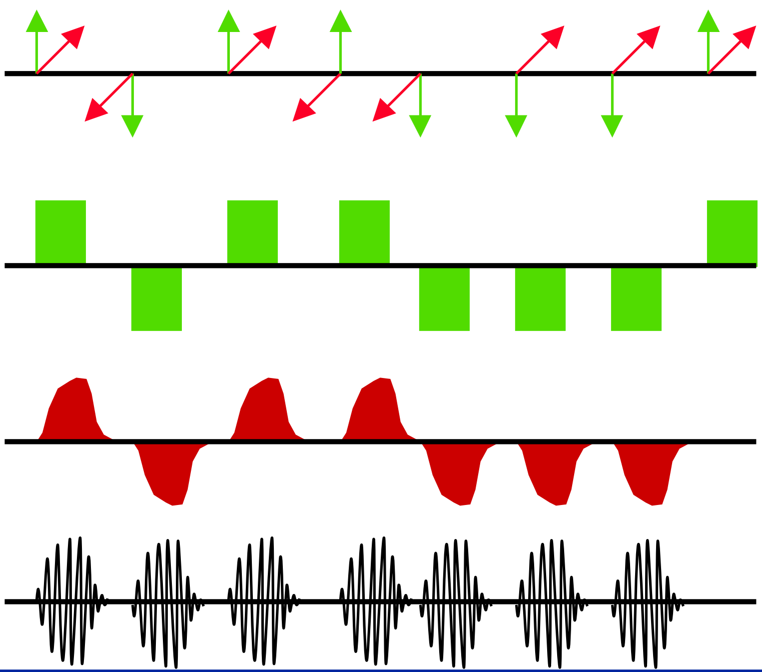
Symbol mapping s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8

I/Q mapping

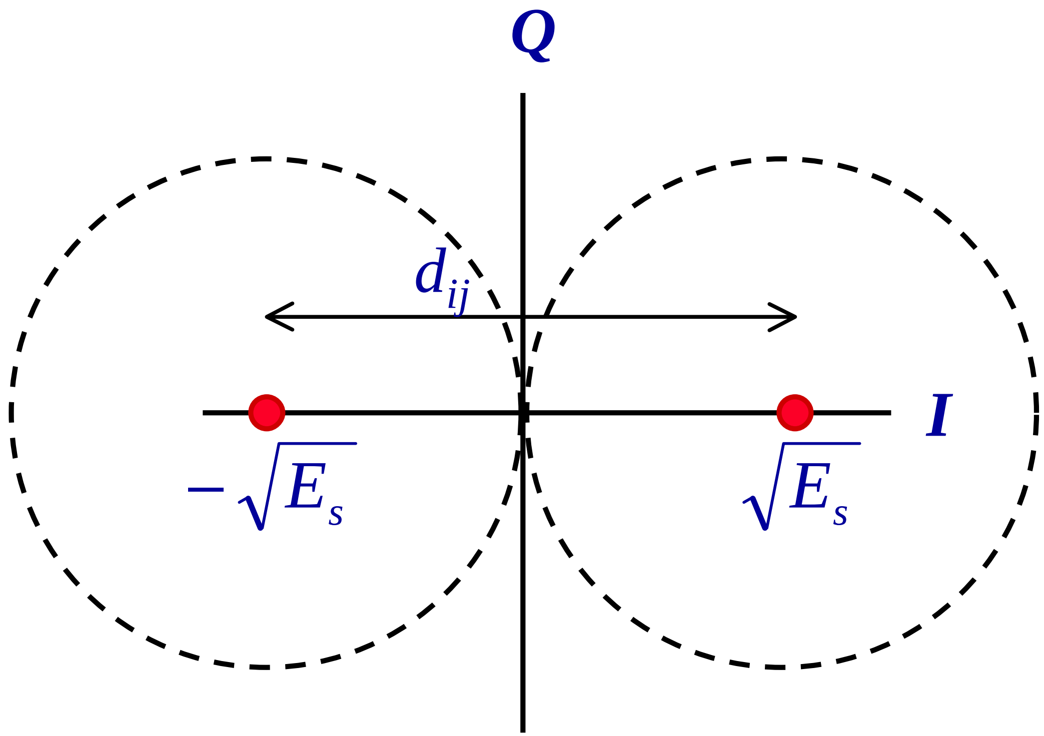
Line coding

Pulse Shaping

RF upconversion



Constellation diagram

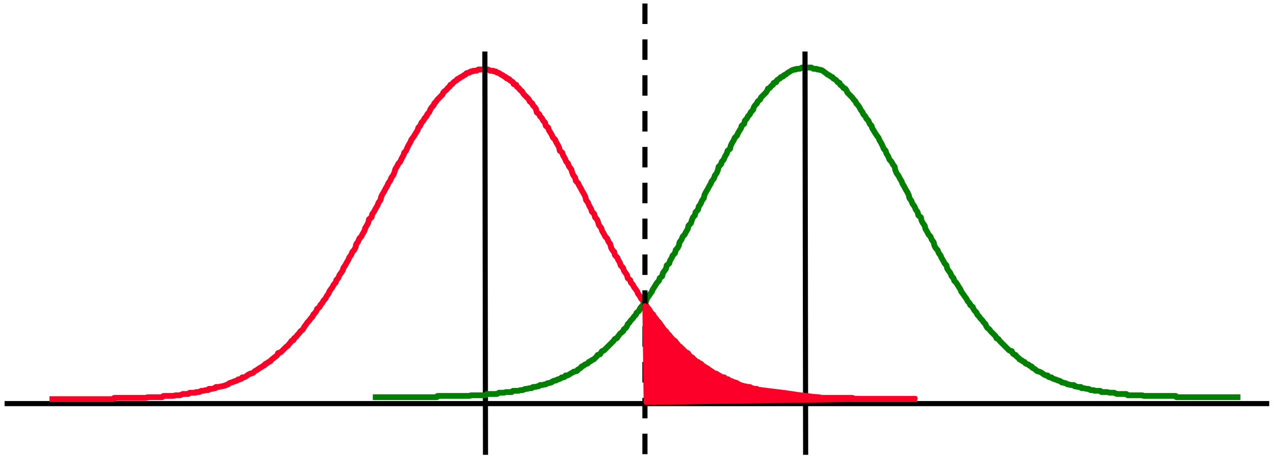


Orthogonal: $i \neq j$

$$\int_0^T \phi_i(t)\phi_j(t)dt = 0$$

Normalize by $\sqrt{2/T_s}$

Error probability



$$P_e = \Pr(z \geq z_0) = \int_{z_0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\, dz = Q(z_0)$$

Error probability

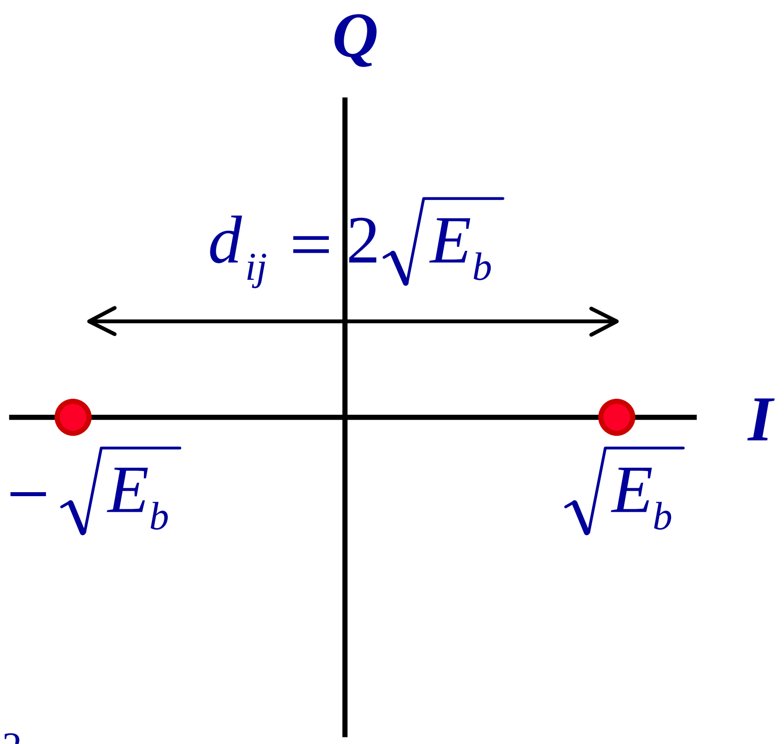
$$z = \frac{A}{\sigma}$$

$$A = \sqrt{2/T_s} \cdot \sqrt{E_s} \qquad A_{norm} = \sqrt{E_s}$$

$$\sigma = \sqrt{N_0 B} \qquad \sigma_{norm} = \sqrt{N_0/2}$$

$$P_e = \Pr(z \geq \frac{d_{ij}}{2}) = Q\left(\frac{A_{norm}}{\sigma_{norm}}\right) = Q\left(\frac{d_{ij}}{\sqrt{2N_0}}\right)$$

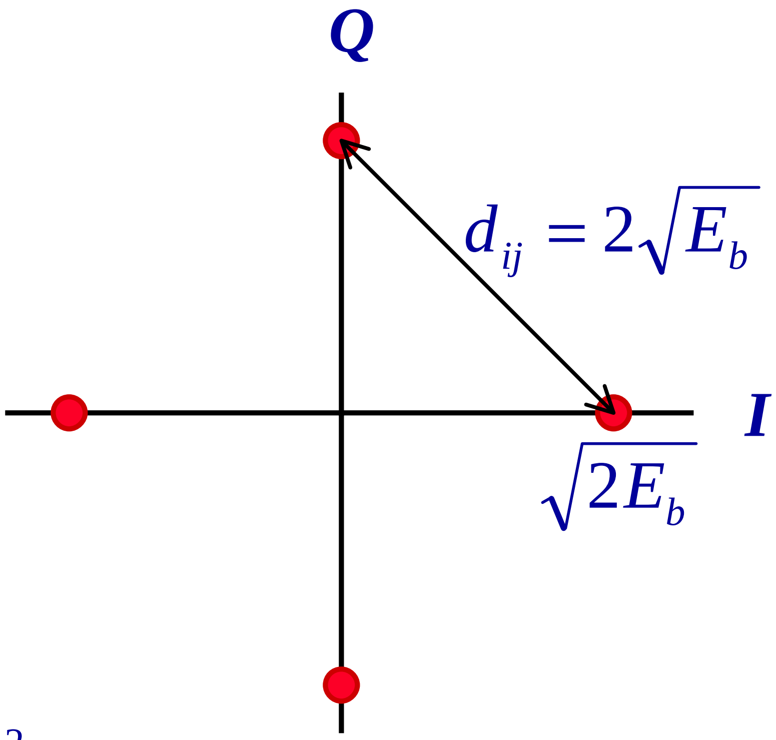
BPSK



$$E_s = E_b = \frac{A^2}{2}$$

$$P_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

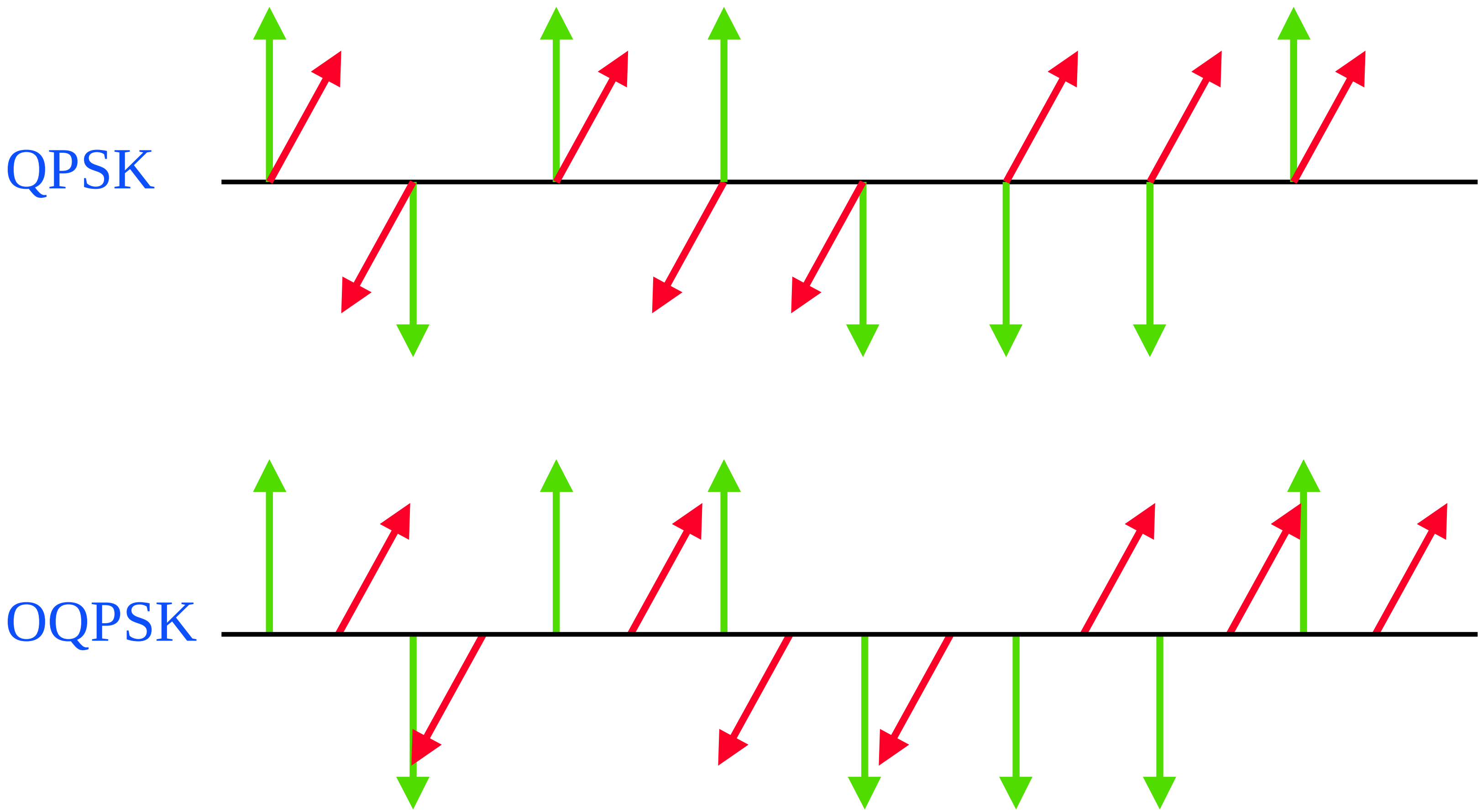
QPSK



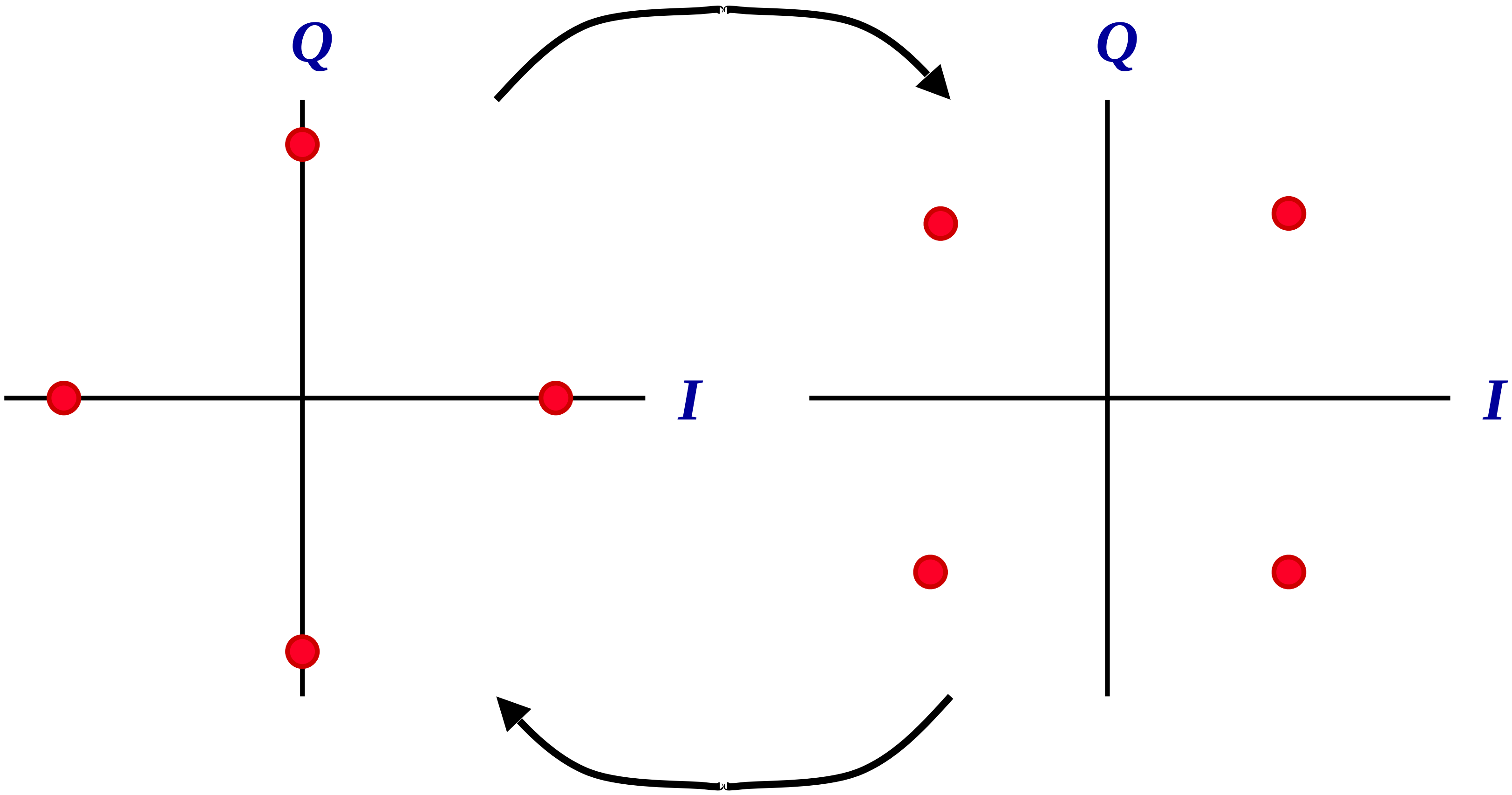
$$E_b = \frac{E_s}{2} = \frac{A^2}{4}$$

$$P_{e,QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

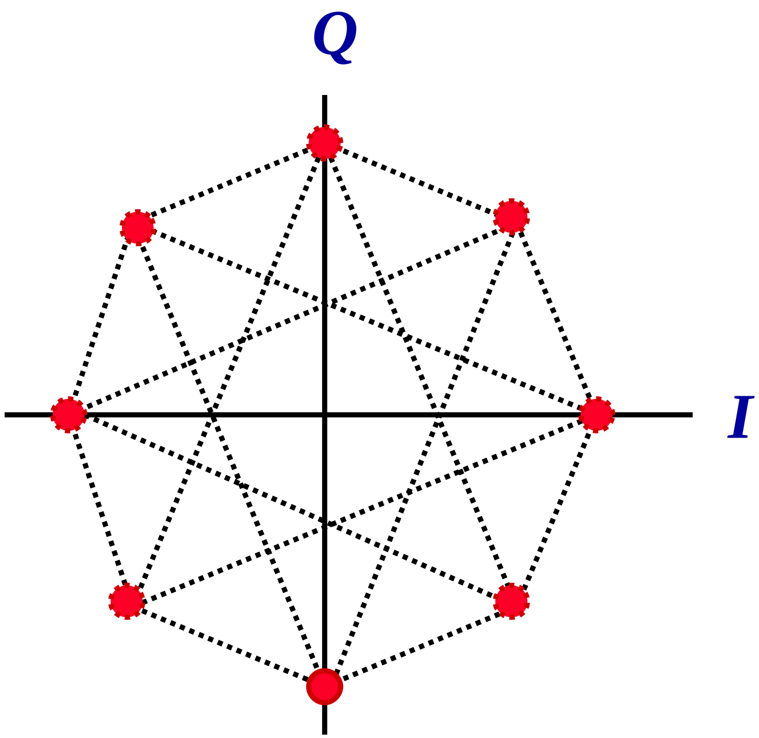
Offset-QPSK



π/4 QPSK

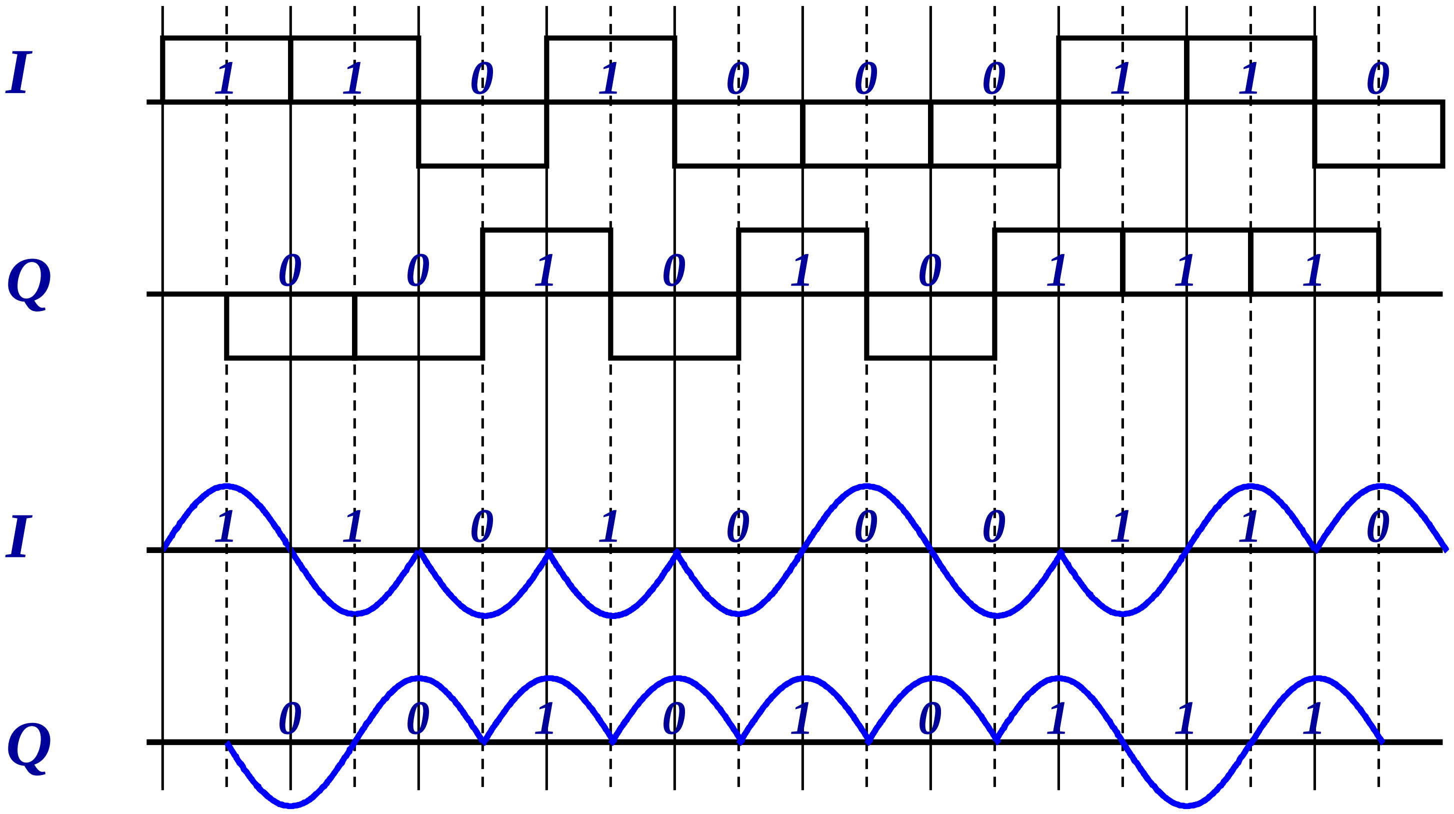


π/4 QPSK



b_k, b_{k-1}	ϕ_k
11	$\pi/4$
01	$3\pi/4$
00	$-3\pi/4$
10	$-\pi/4$

Minimum Shift Keying



MSK is cos-shaped OQPSK

Minimum Shift Keying

$$S_{MSK} = m_I(t)\cos\left(\frac{\pi t}{2T_b}\right)\cos(2\pi f_c t) \quad + \quad m_Q(t)\sin\left(\frac{\pi t}{2T_b}\right)\sin(2\pi f_c t)$$

$$S_{MSK} = \cos(2\pi[f_c + m(t)\Delta f]t)$$

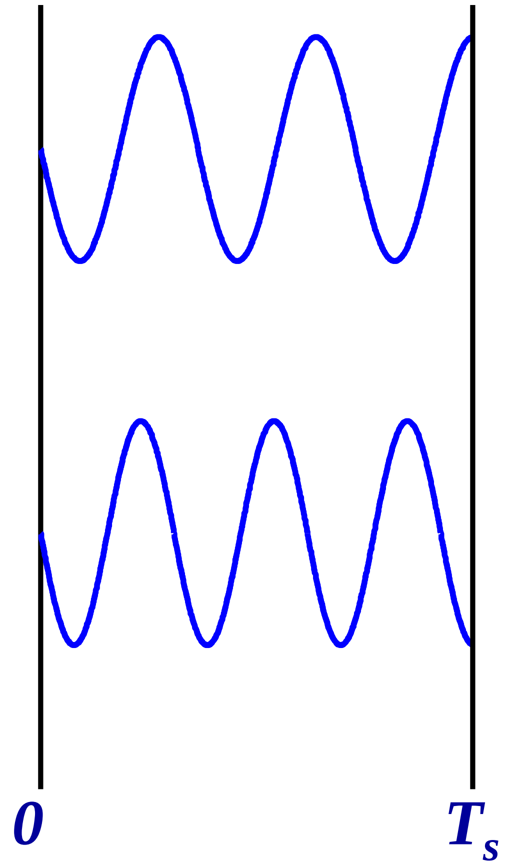
FSK with $\Delta f = 1/4T_b$

Continuous phase FSK (CPFSK)
Continuous phase modulation (CPM)

Minimum Shift Keying

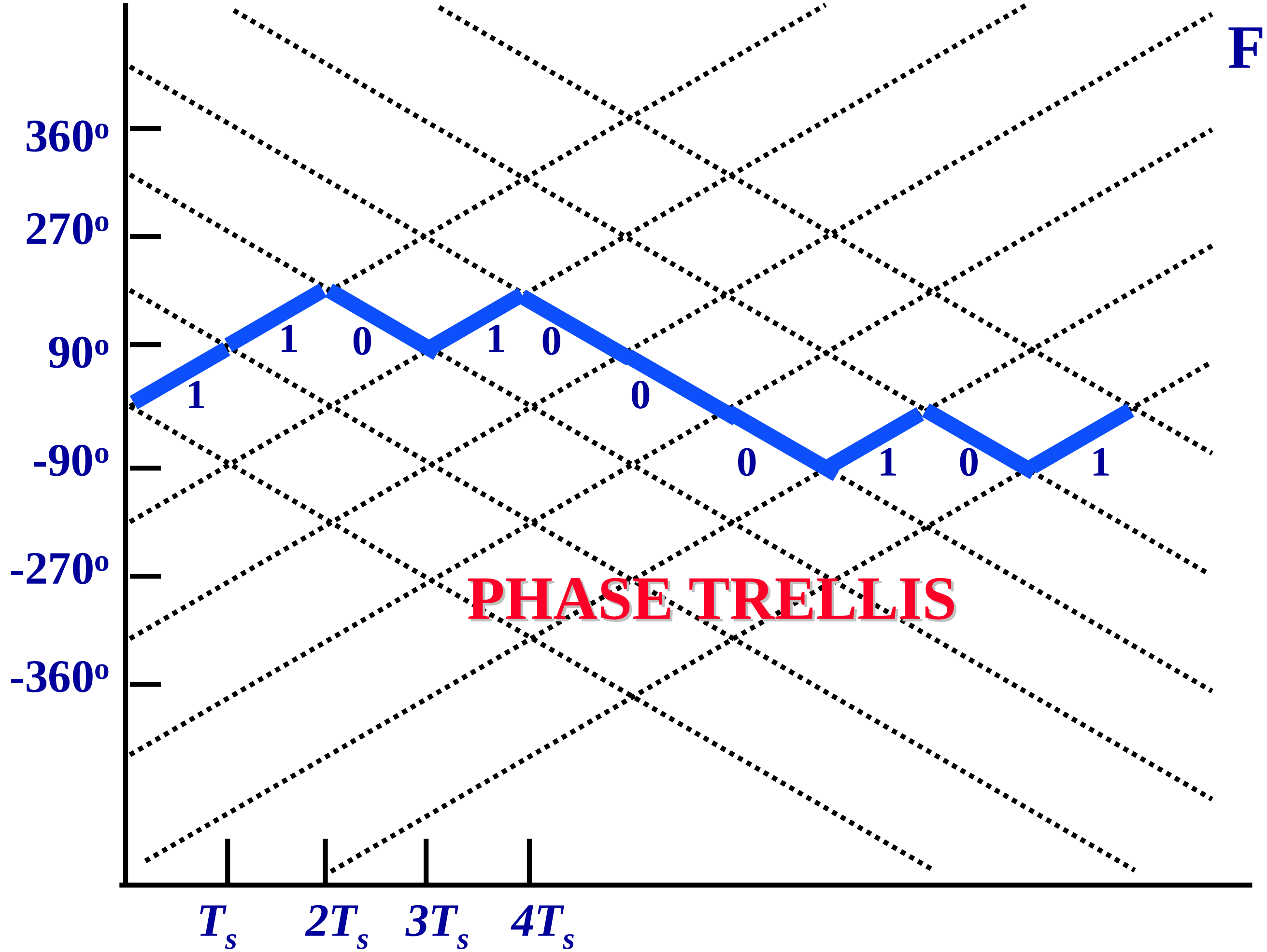
$$0: \quad f = f_c - \frac{1}{4T_s}$$

$$1: \quad f = f_c + \frac{1}{4T_s}$$



Orthogonal frequency keying

Minimum Shift Keying

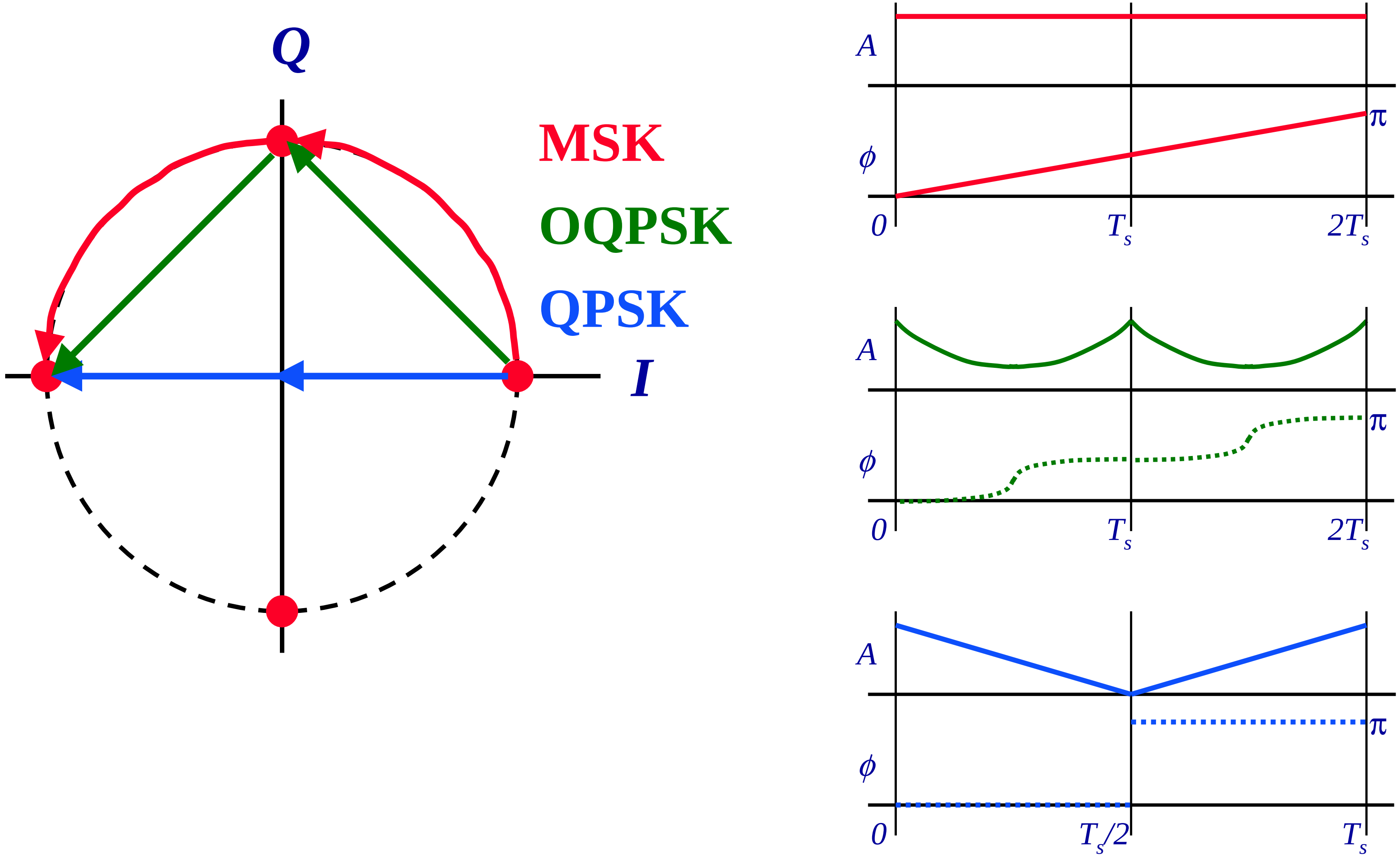


FSK:

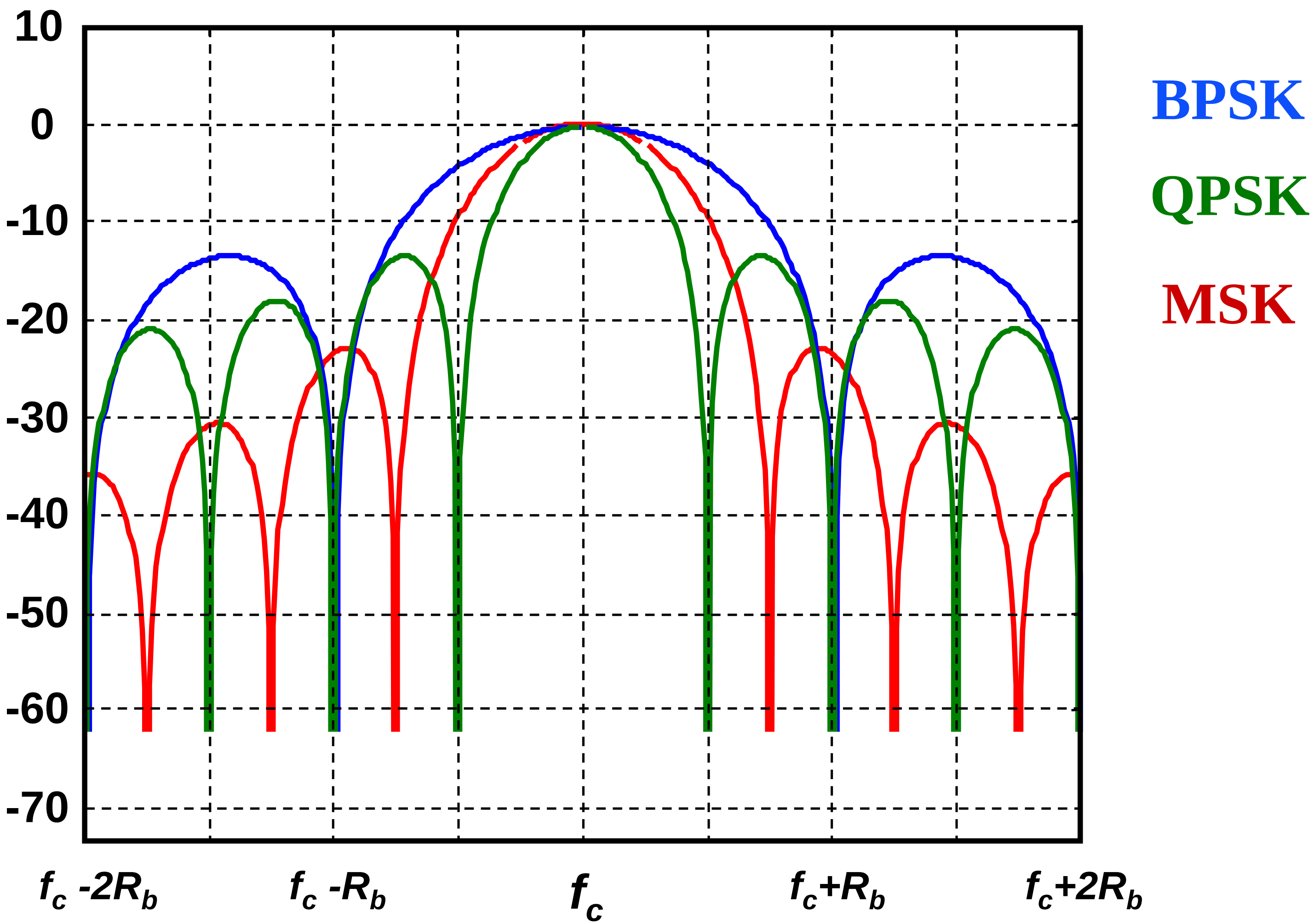
$$1: \quad f = f_c + \frac{1}{4T_s}$$

$$0: \quad f = f_c - \frac{1}{4T_s}$$

Symbol transistions

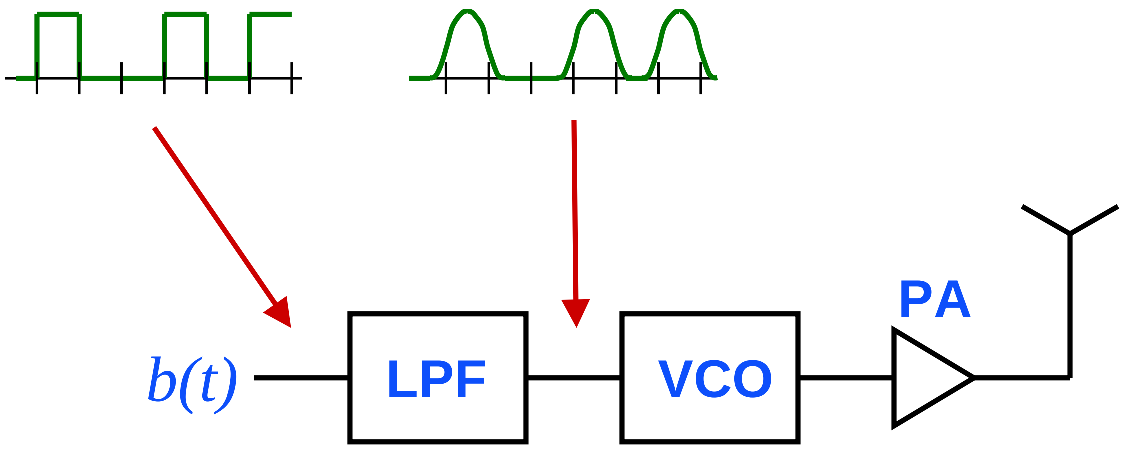


Power spectral densities



GMSK

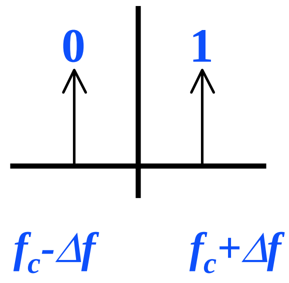
- **Gaussian shaping of phase signal**

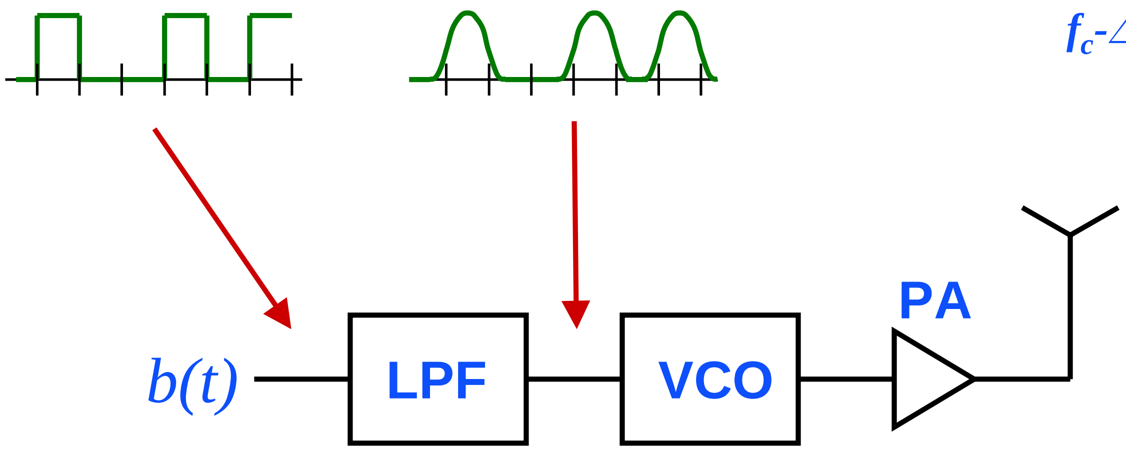


$$P_{e,GMSK} = Q\left(\sqrt{\frac{2\alpha E_b}{N_0}}\right) \qquad a = \begin{cases} 0.68 & BT = 0.25 \\ 0.85 & BT \to \infty \end{cases}$$

FSK

- **logical "1": positive frequency shift**
- **logical "0": negative frequency shift**



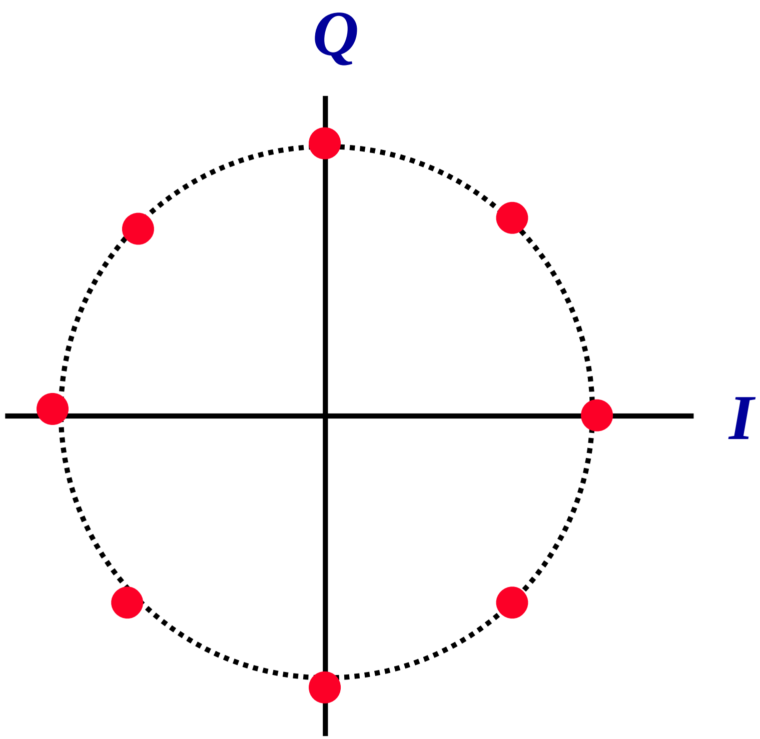


$$P_{e,FSK} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$B_T = 2(\Delta f + R)$$

Other modulation schemes

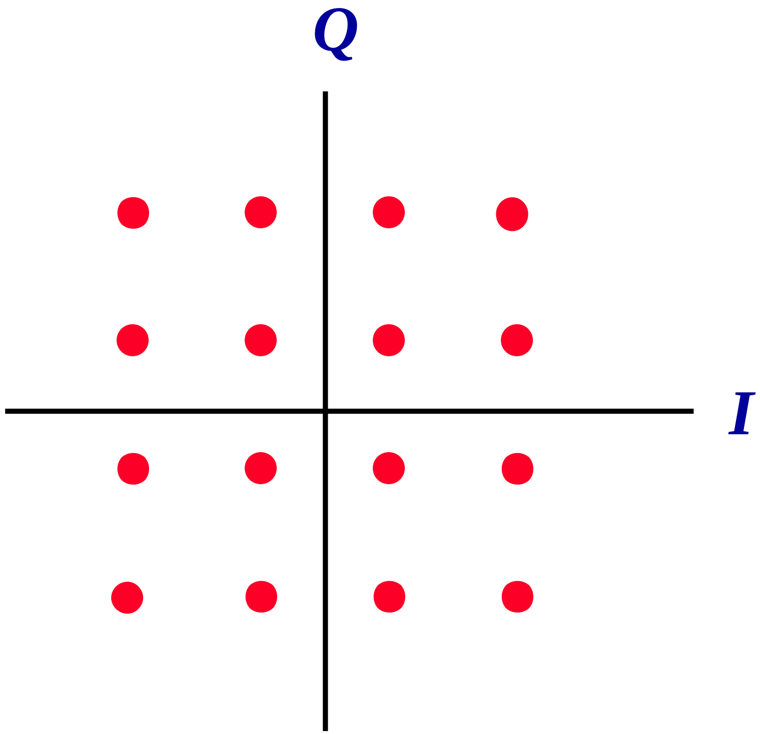
***M*-ary PSK:**



M	2	4	8	16	32	64
η_b $E_b/N0$	0.5 10.5	1 10.5	1.5 14	2 18.5	2.5 23.4	3 28.5

Other modulation schemes

***M*-ary QAM:**



M	4	16	64	256	1024	4096
η_b	1	2	3	4	5	6
$E_b/N0$	10.5	15	18.5	24	28	33.5

Digital transmission chain

Information bits

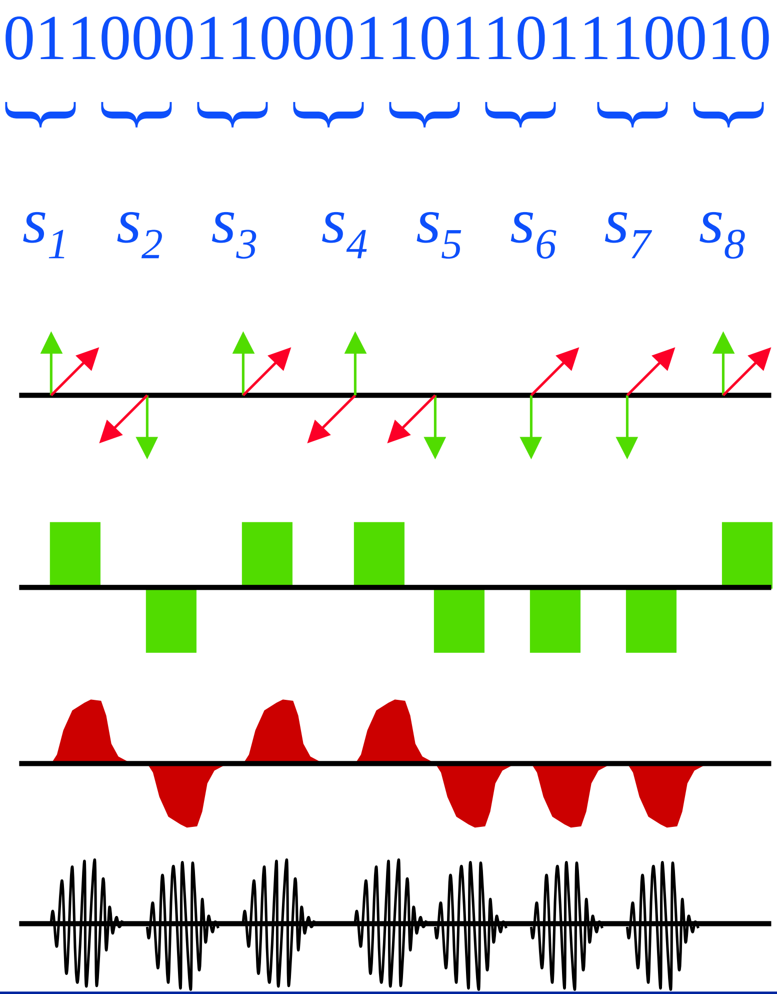
Symbol mapping

I/Q mapping

→ **Line coding**

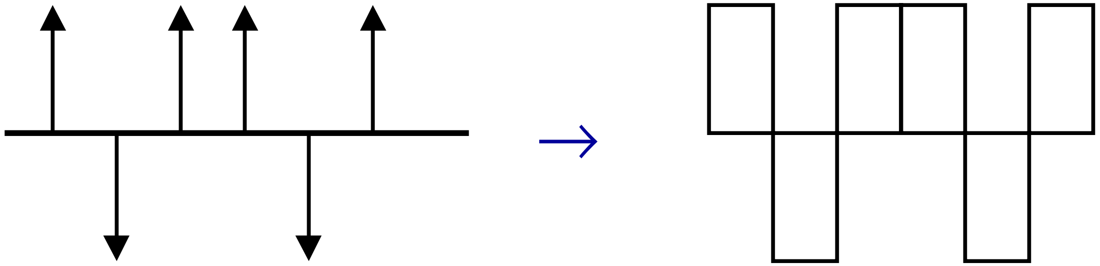
Pulse Shaping

RF upconversion



Line coding

impulse train → pulse train

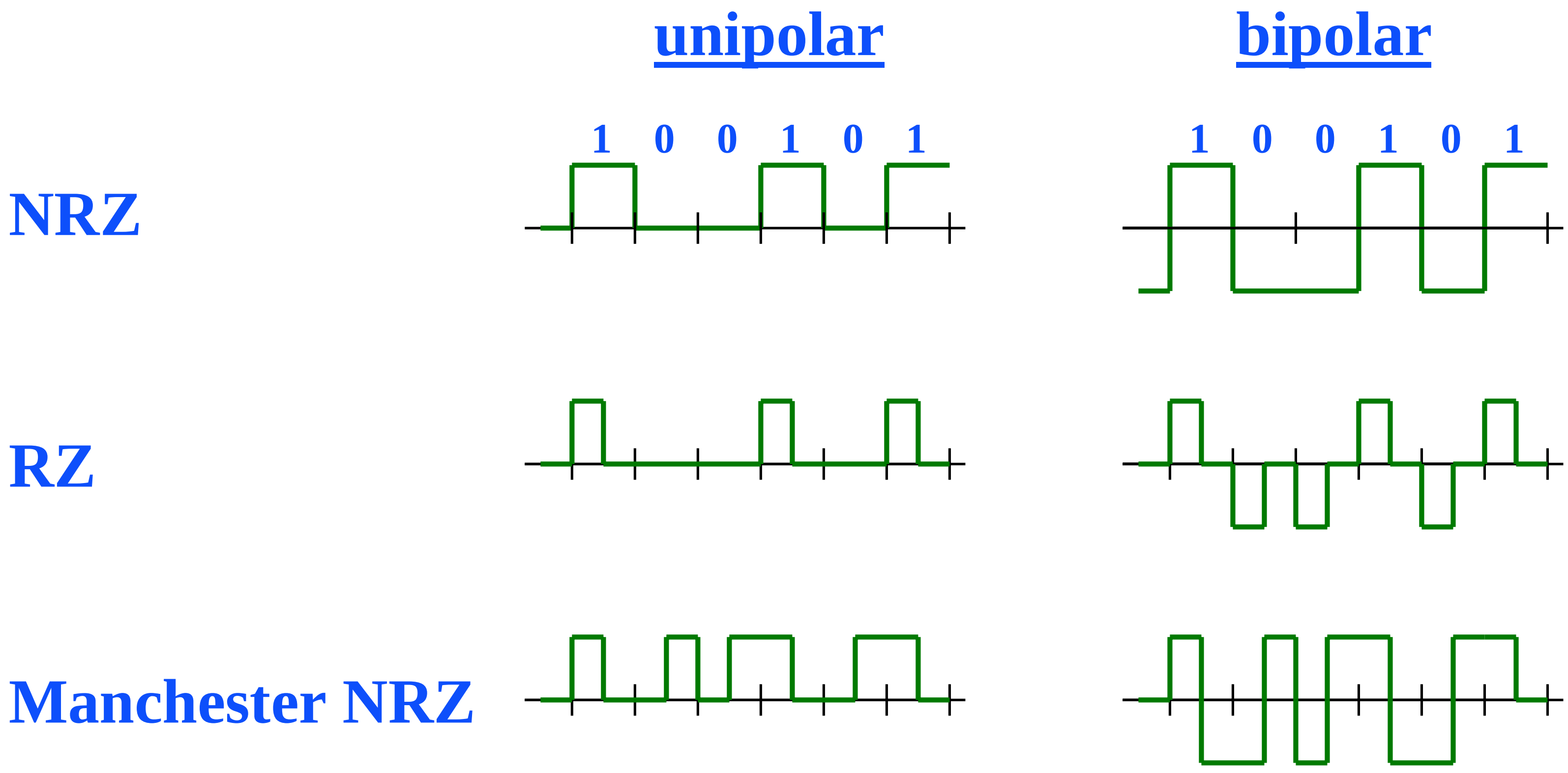


- **Spectral widening**
- **bit synchronization**
- **DC component**

- **unipolar**
- **bipolar**

- **return-to-zero (RZ)**
- **non-return-to-zero (NRZ)**
- **Manchester**

Line coding formats



Digital transmission chain

Information bits

Symbol mapping

I/Q mapping

Line coding

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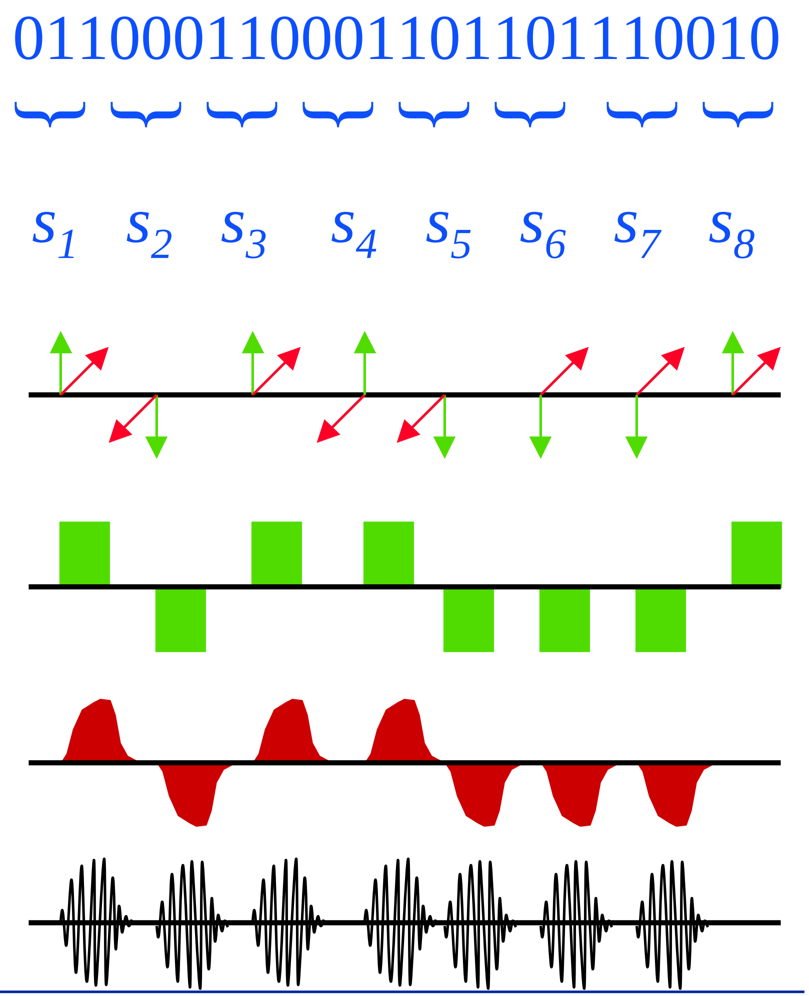
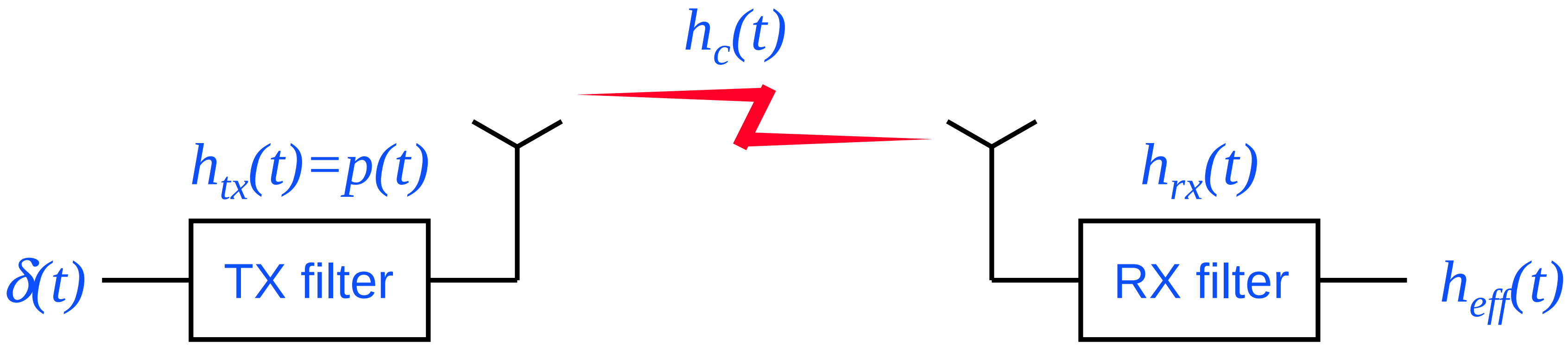


Pulse shaping

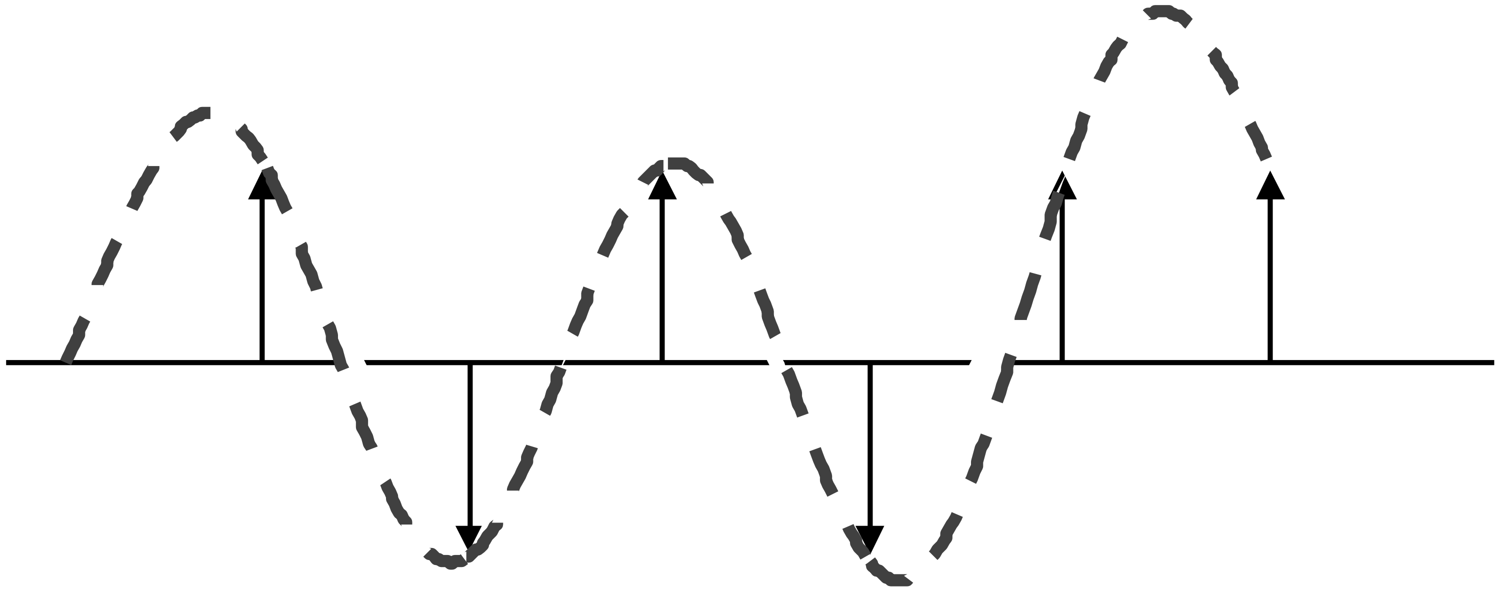


$$h_{eff}(t) = \delta(t) * h_{tx}(t) * h_c(t) * h_{rx}(t)$$

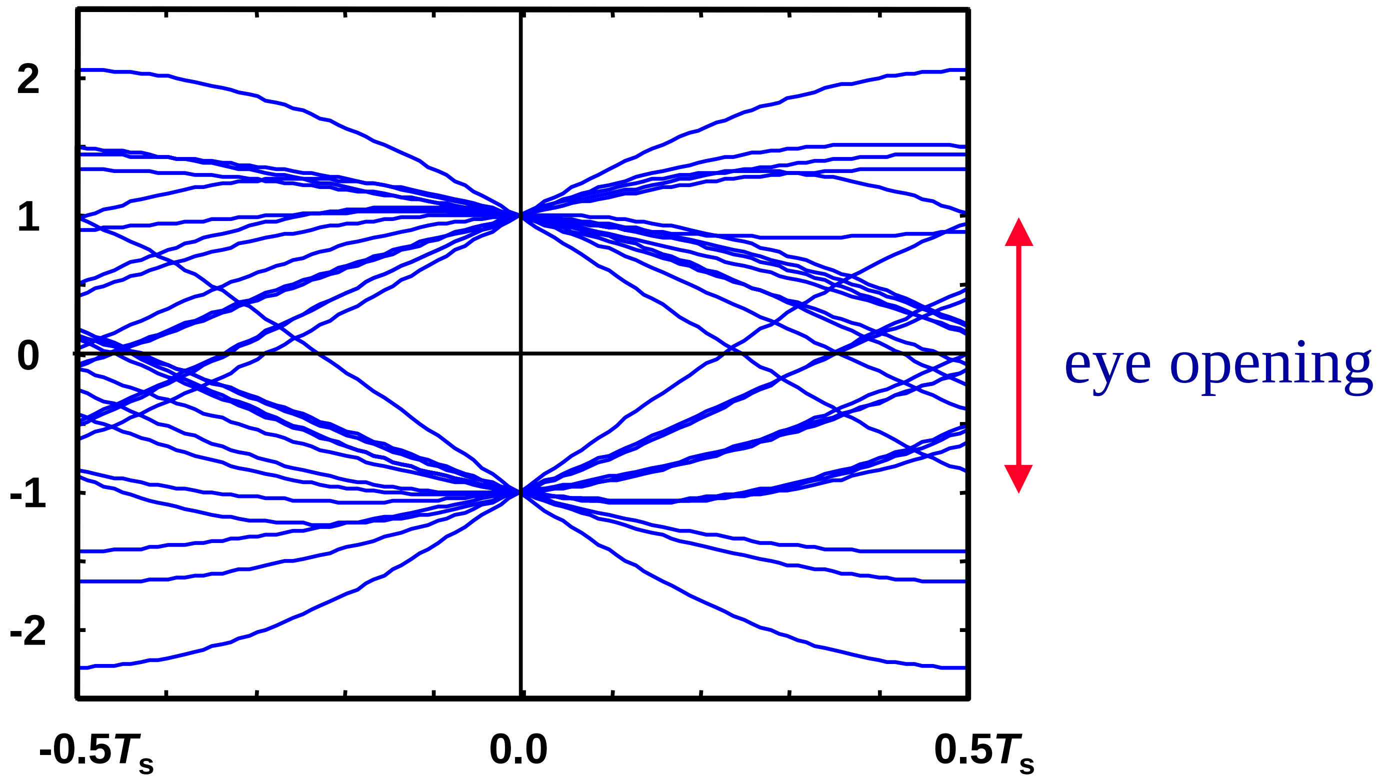
Nyquist criterion:

$$h_{eff}(nT_s) = \begin{cases} K & n = 0 \\ 0 & n \neq 0 \end{cases}$$

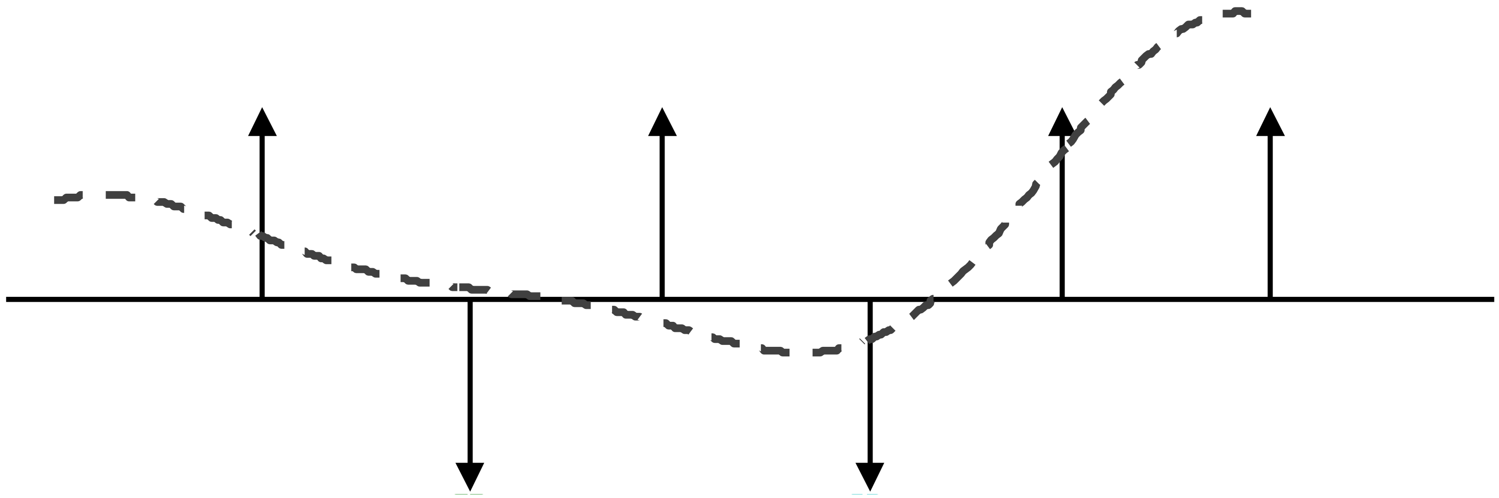
Inter-symbol interference



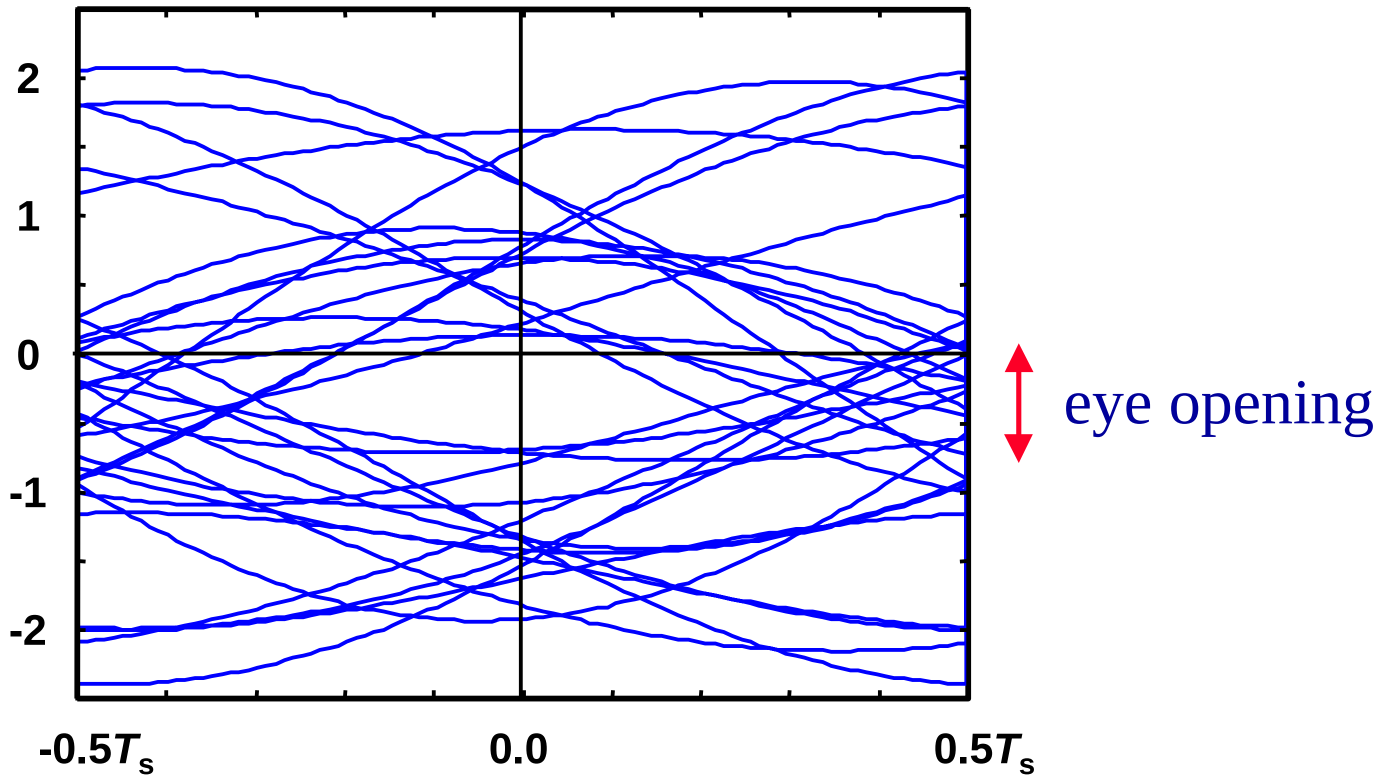
Eye diagram



Inter-symbol interference



Eye diagram



Pulse shaping

Choose optimal $h_{tx}(t)$ such that:

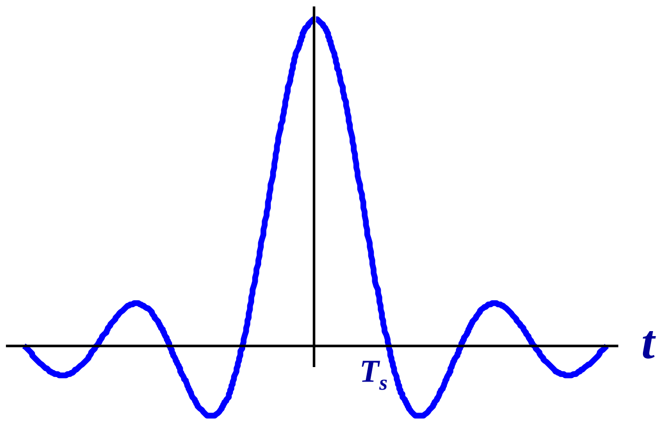
- spectral efficiency is optimized (minimal BW, low leakage)
- bit synchronization is facilitated
- ISI effects are minimized

Choose optimal $h_{eff}(t)=h_{tx}(t)*h_{rx}(t)$ such that:

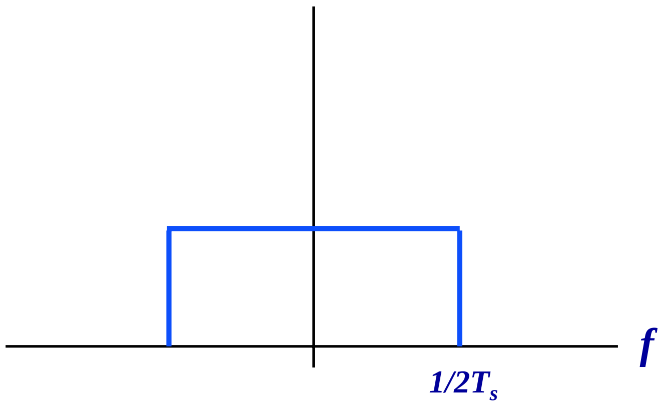
- bit synchronization is optimized
- ISI effects are minimized

Sinc pulse

$$h_{eff}(t)=\frac{\sin(\pi t/T_s)}{(\pi t)/T_s} \quad \overset{F}{\leftrightarrow} \quad H_{eff}(f)=\frac{1}{f_s}\Pi\left(\frac{f}{f_s}\right)$$



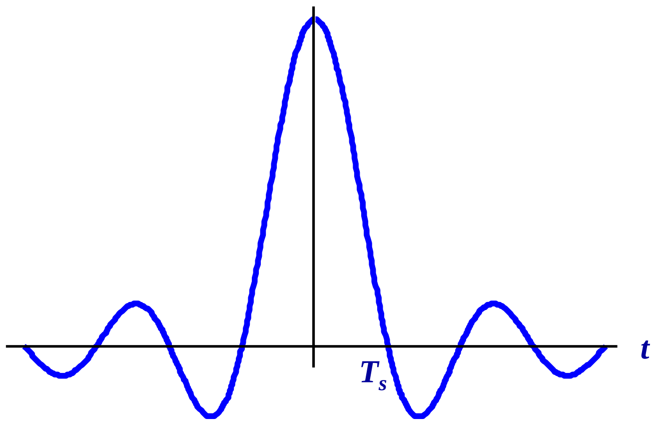
$\overset{F}{\leftrightarrow}$



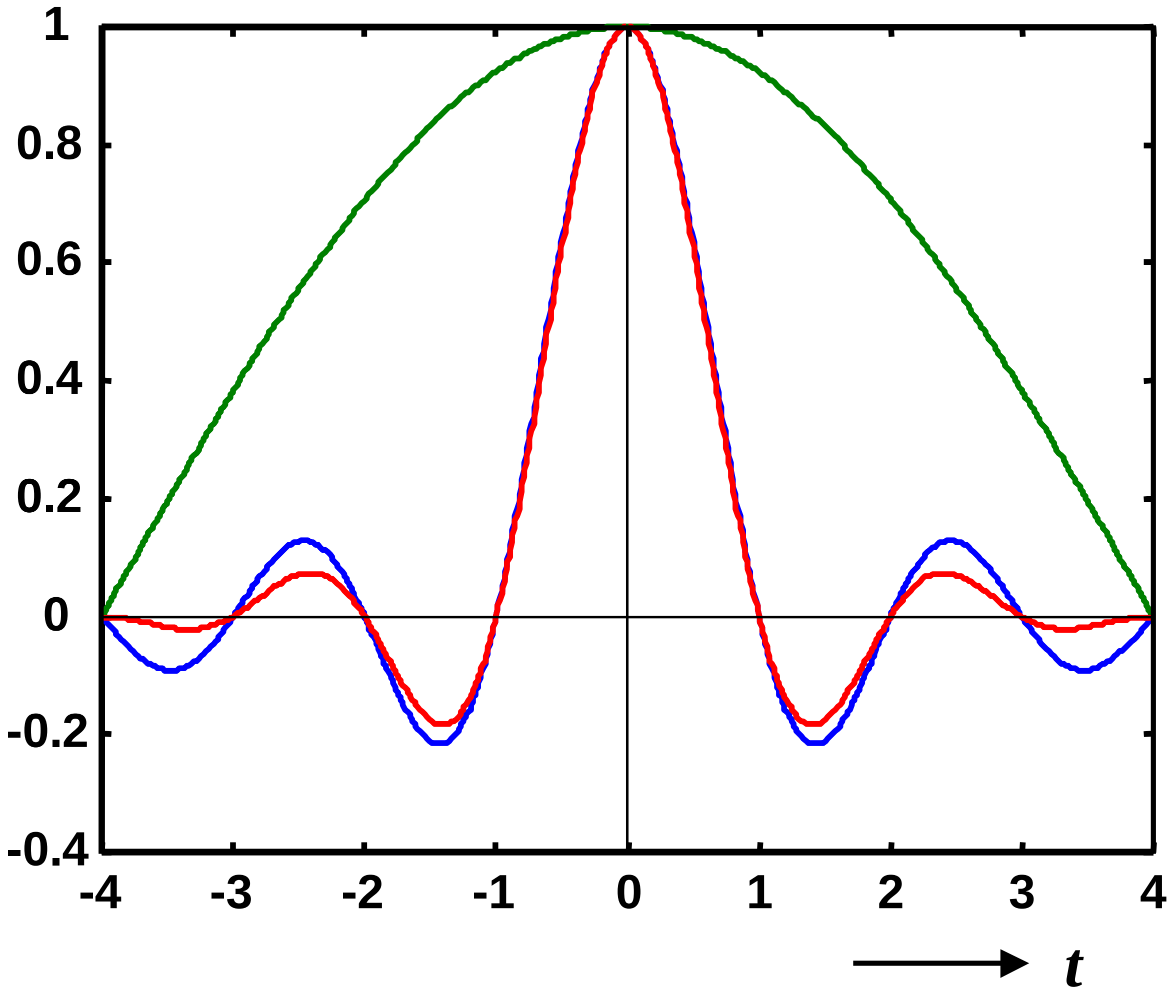
Shaping

Nyquest criterion remains satisfied for *z(t)*:

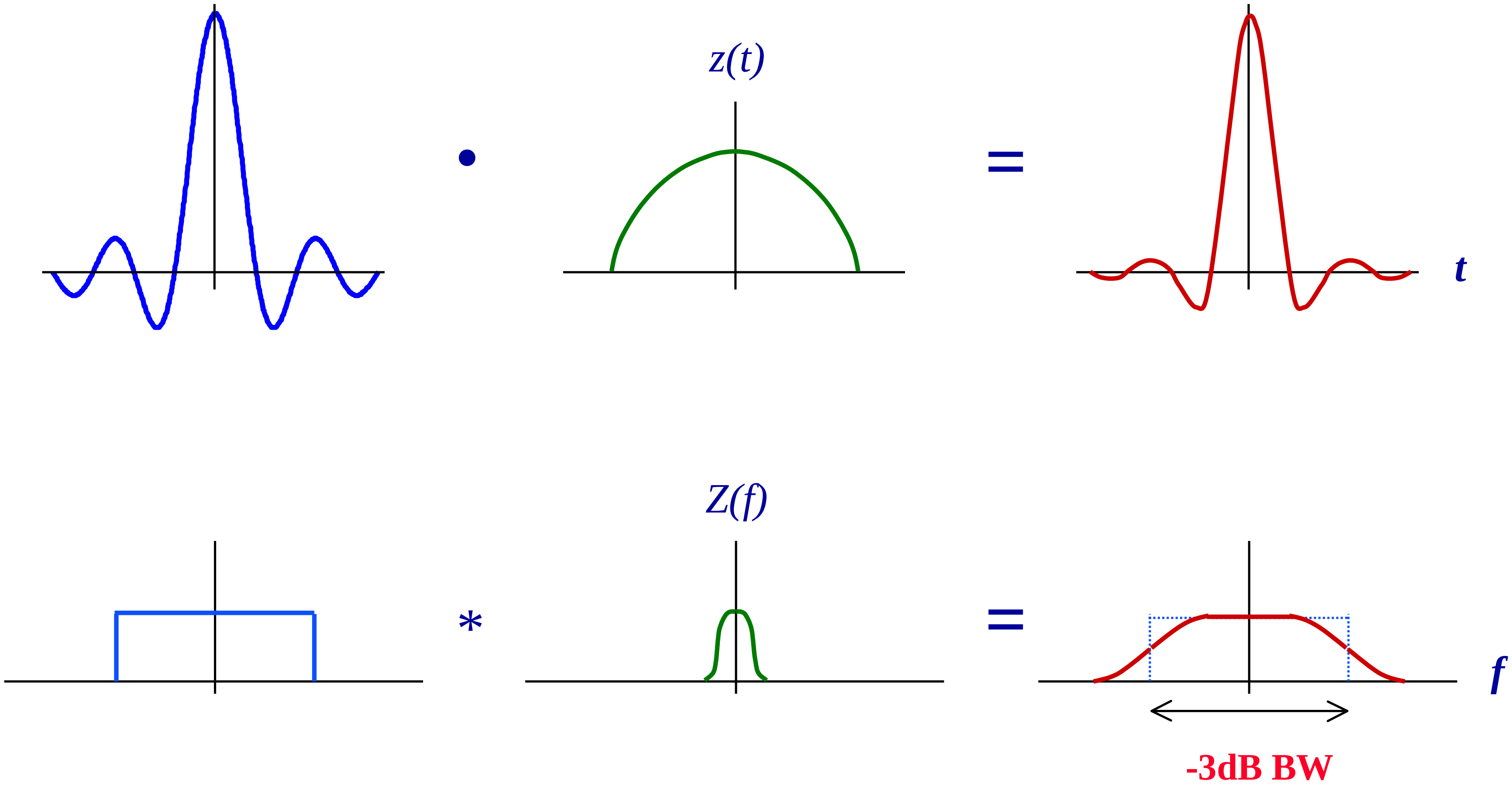
$$h_{eff}(t) = \frac{\sin(\pi t / T_s)}{(\pi t)/T_s} \bullet z(t)$$



Shaping



Shaping



Raised cosine shaping

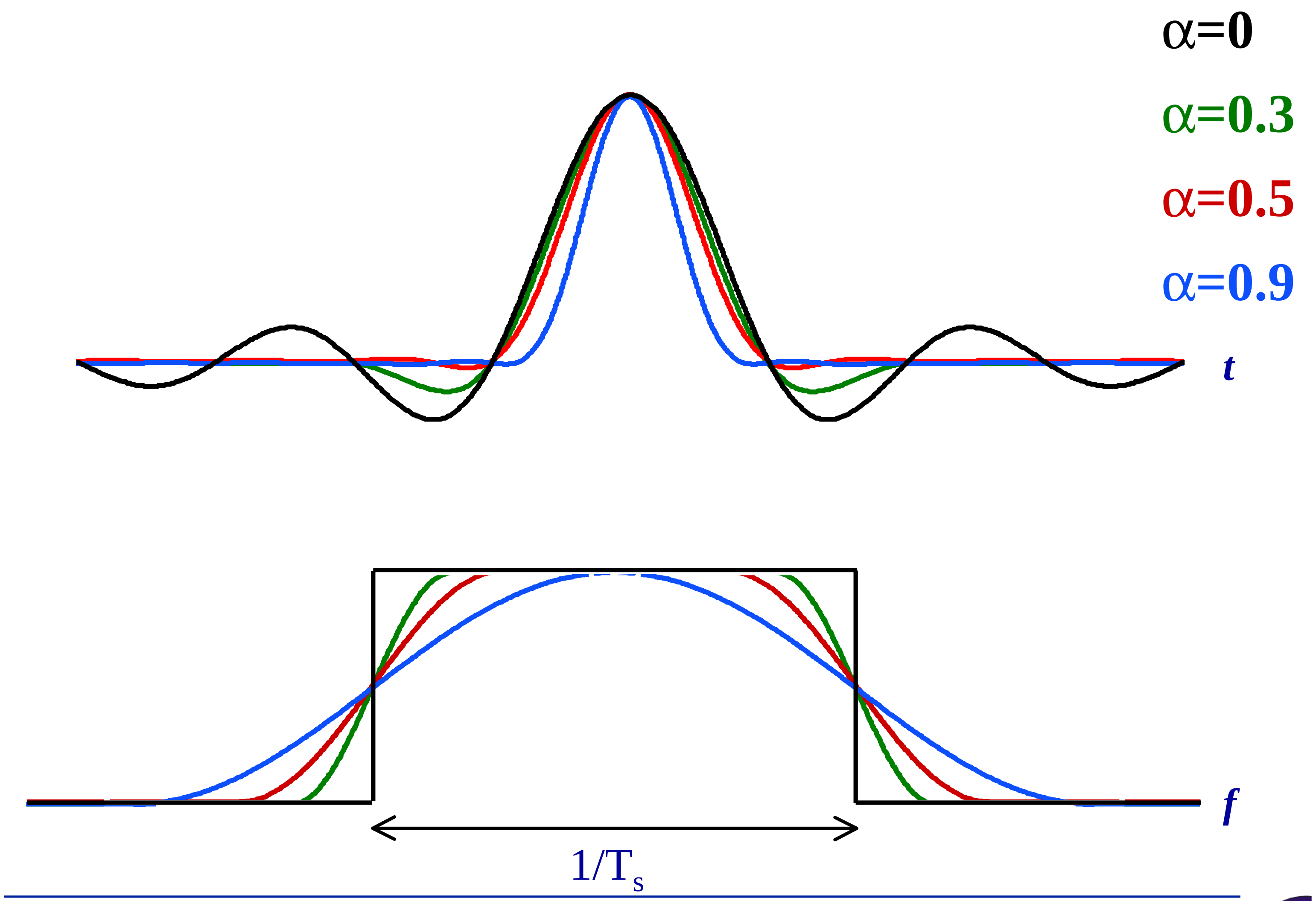
$$H_{eff}(f) = T_s \qquad 0 \le |f| \le (1-\alpha)/2T_s$$

$$= 0 \qquad |f| \ge (1+\alpha)/2T_s$$

$$= \frac{T_s}{2}\left[1 + \cos\left(\frac{\pi T_s}{\alpha}\left\{f - \frac{1-\alpha}{2T_s}\right\}\right)\right] \qquad \text{rest}$$

$$h_{eff}(t) = \left[\frac{\cos(2\pi\alpha\, t/T_s)}{1-(4\alpha\, t/T_s)^2}\right] \cdot \left[\frac{\sin(\pi t/T_s)}{\pi t/T_s}\right]$$

Raised cosine shaping



RRC shaping

$h_{eff}(t)=h_{tx}(t)*h_{rx}(t)$ **Raised Cosine**

$h_{tx}(t)=h_{rx}(t)=sqrt(h_{eff}(t))$ **Root Raised Cosine**

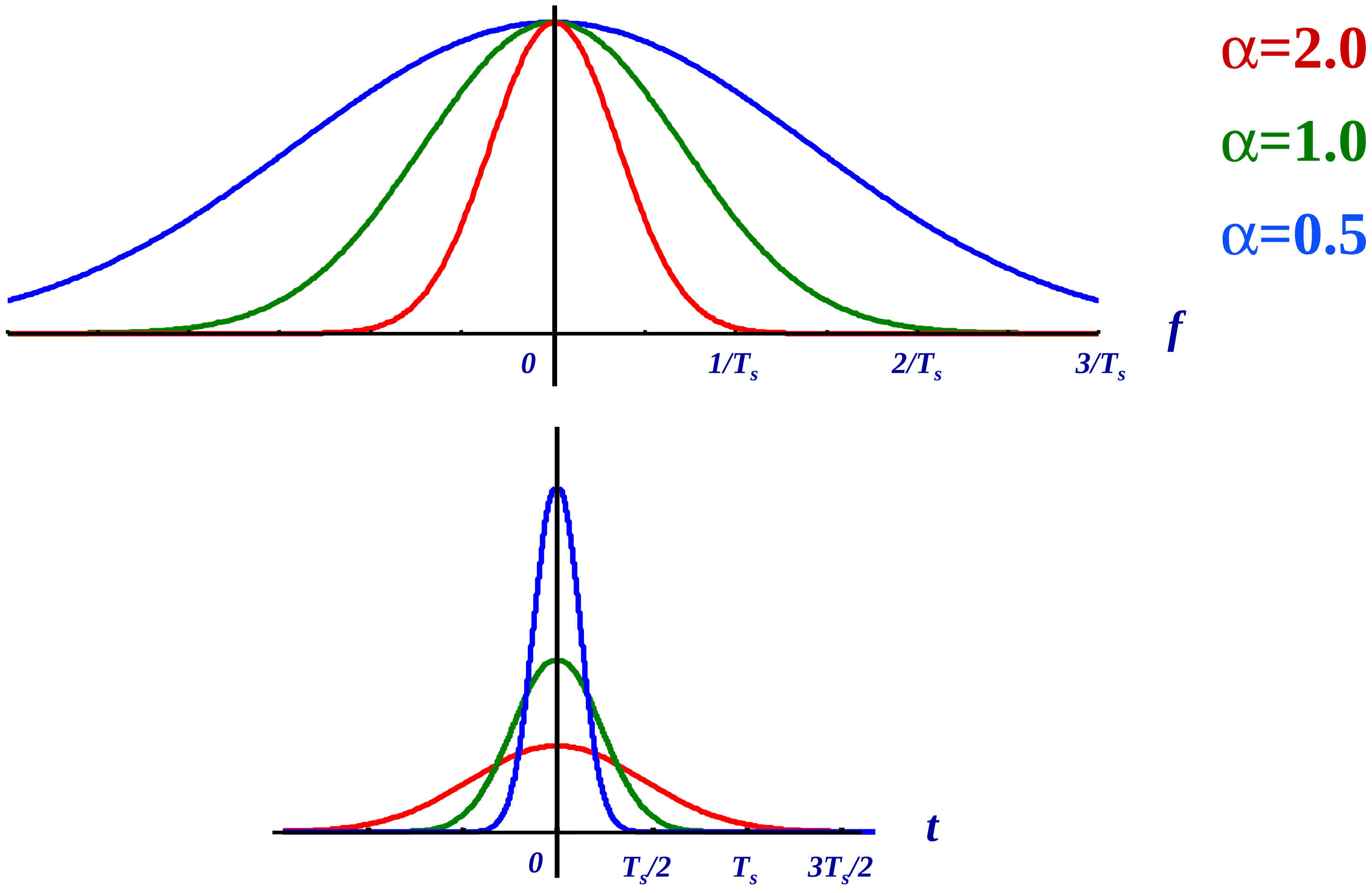
Gaussian shaping

$$H_{eff}(f) = \exp(-\alpha^2 f^2 T_s^2)$$

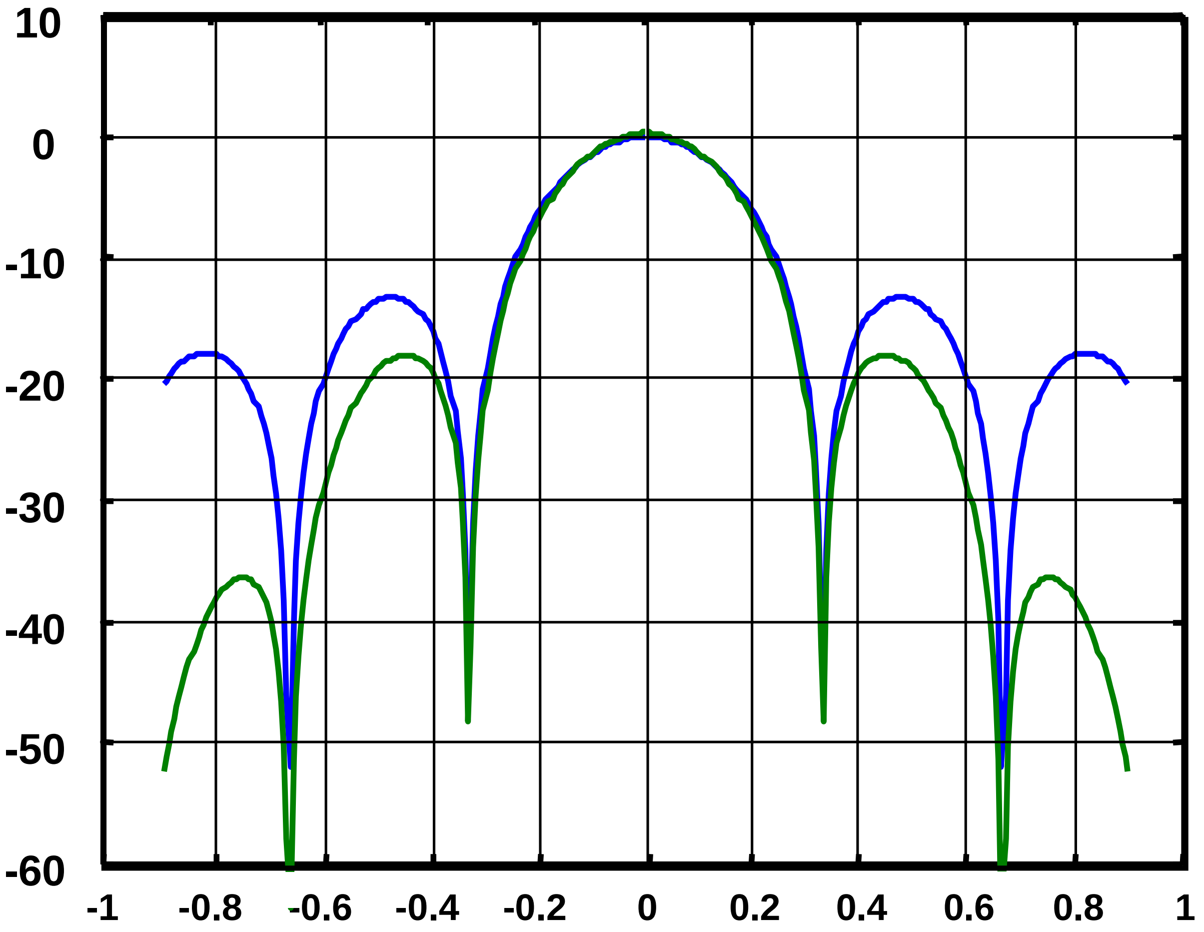
$$h_{eff}(t) = \frac{\sqrt{\pi}}{\alpha}\exp(-\frac{\pi^2}{\alpha^2}\frac{t}{T_s^2}{}^{2})$$

$$B_{-3dB} = \frac{\sqrt{\ln(2)/2}}{\alpha T_s}$$

Gaussian shaping



Spectral effect



Digital transmission chain

Information bits 0110001100011011101110010

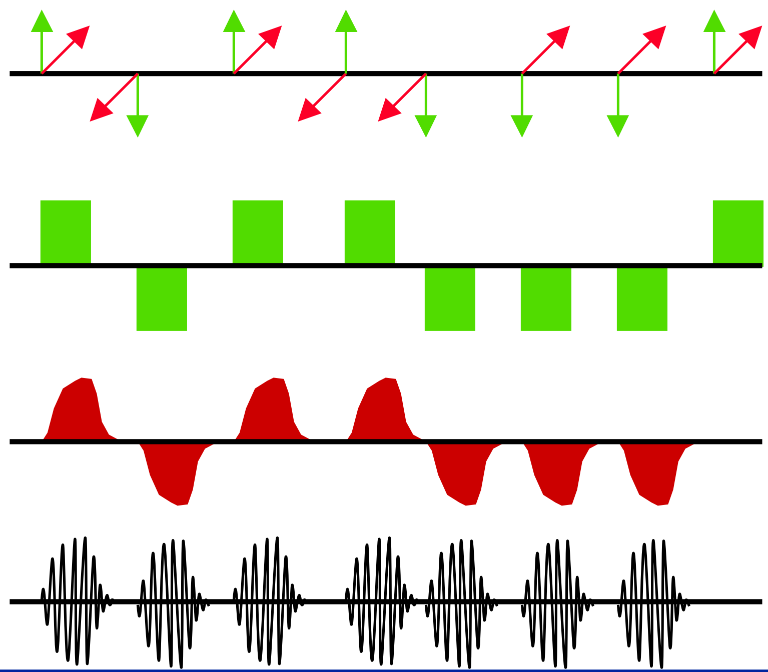
Symbol mapping s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8

I/Q mapping

Line coding

Pulse Shaping

RF upconversion

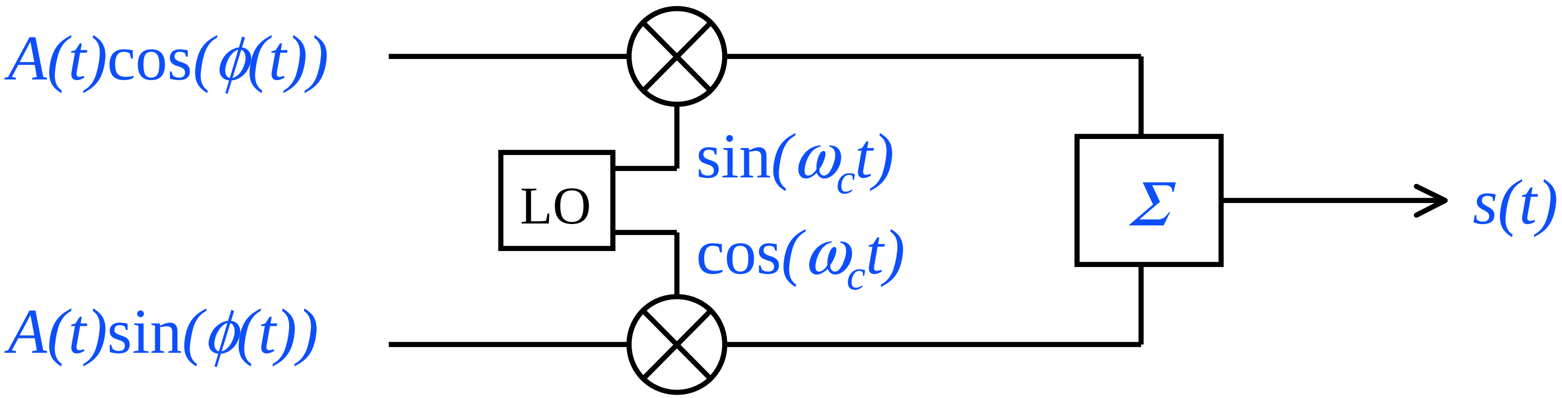


RF upconversion

$$s(t) = A(t)\cos(\omega_c t + \phi(t))$$

Via I and Q modulation:

$$s(t) = A(t)[\cos(\phi(t))\sin(\omega_c t) - \sin(\phi(t))\cos(\omega_c t)]$$



RF upconversion

$$s(t) = A(t)\cos(\omega_c t + \phi(t))$$

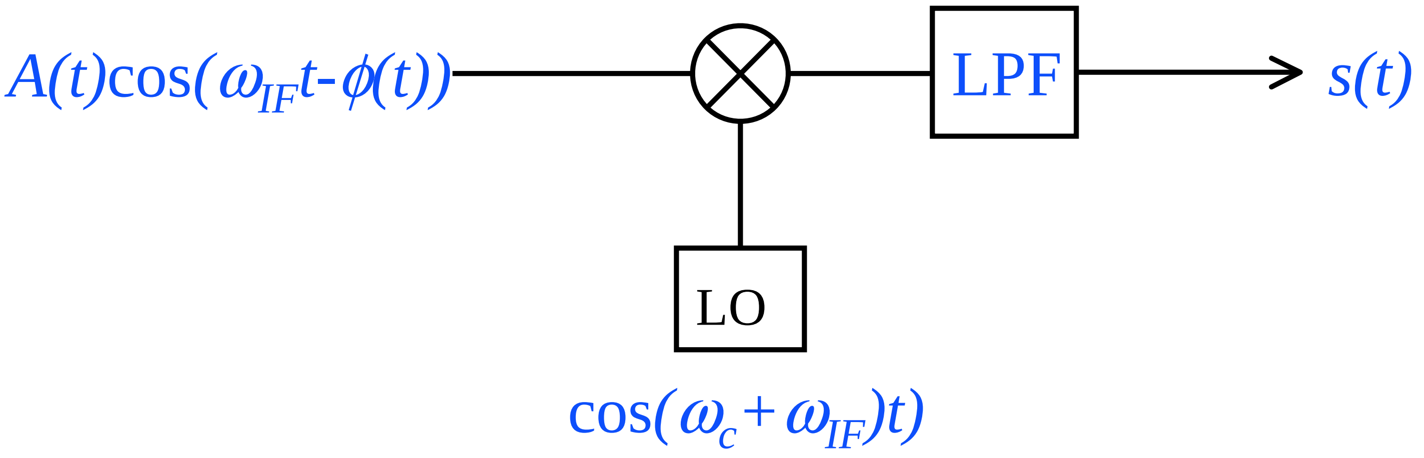
Via IF stage: $s_{IF}(t) = A(t)\cos(\omega_{IF} t - \phi(t))$

$$s(t) = LPF\{2 \cdot s_{IF}(t) \cdot \cos((\omega_c + \omega_{IF})t)$$

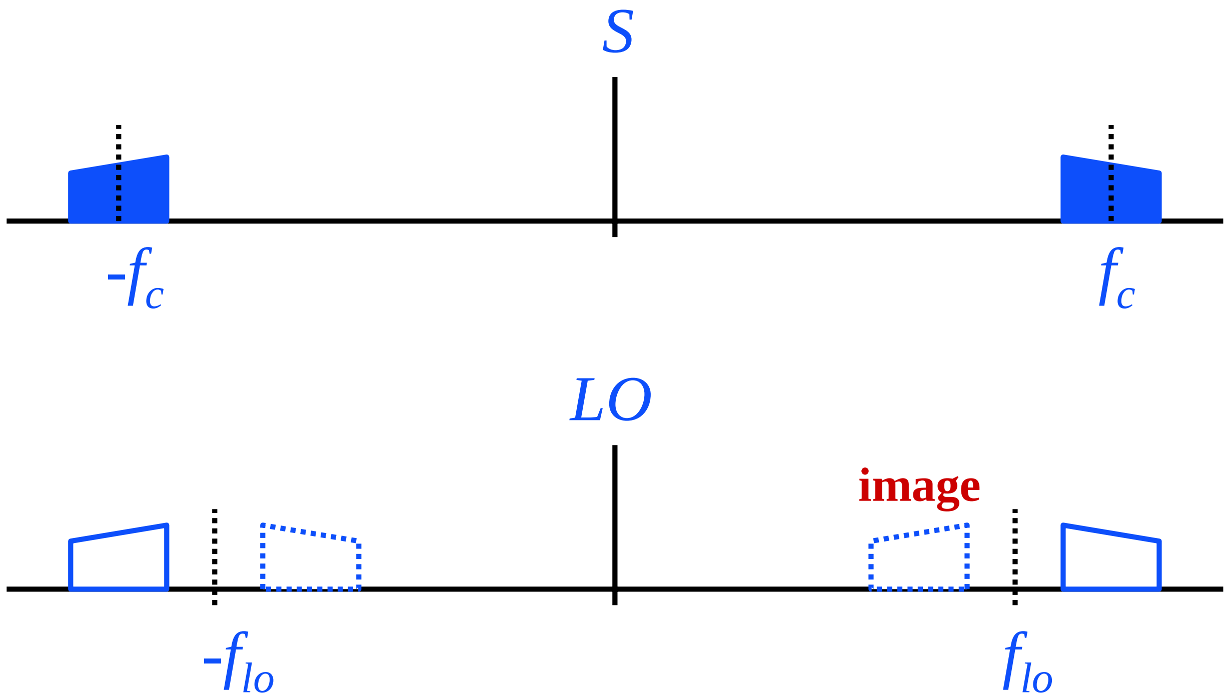
$$= LPF\{A(t)[\cos(\omega_c t + \phi(t)) + A(t)\cos((\omega_c + 2\omega_{IF})t - \phi(t))]\}$$

RF upconversion

Via IF stage:



RF downconversion



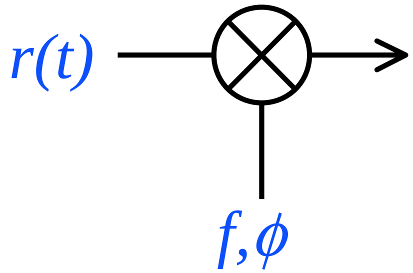
RF downconversion

- **High IF:** $f_{lo} - f_c >> W_{band}$
- **Low IF:** $f_{lo} - f_c \approx W_{band}$
- **Zero IF:** $f_{lo} - f_c = 0$

Coherent versus non-coherent reception

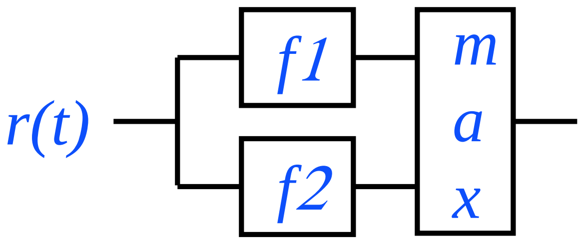
Coherent:

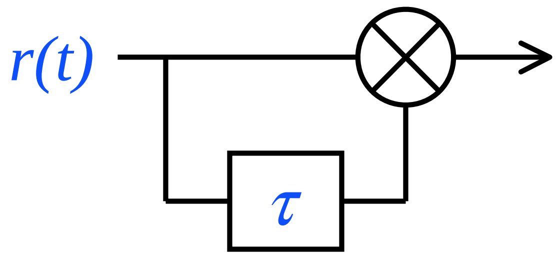
- phase reference
- phase and frequency are retrieved



Non-coherent:

- energy detection
- differential detection (previous symbol is reference)



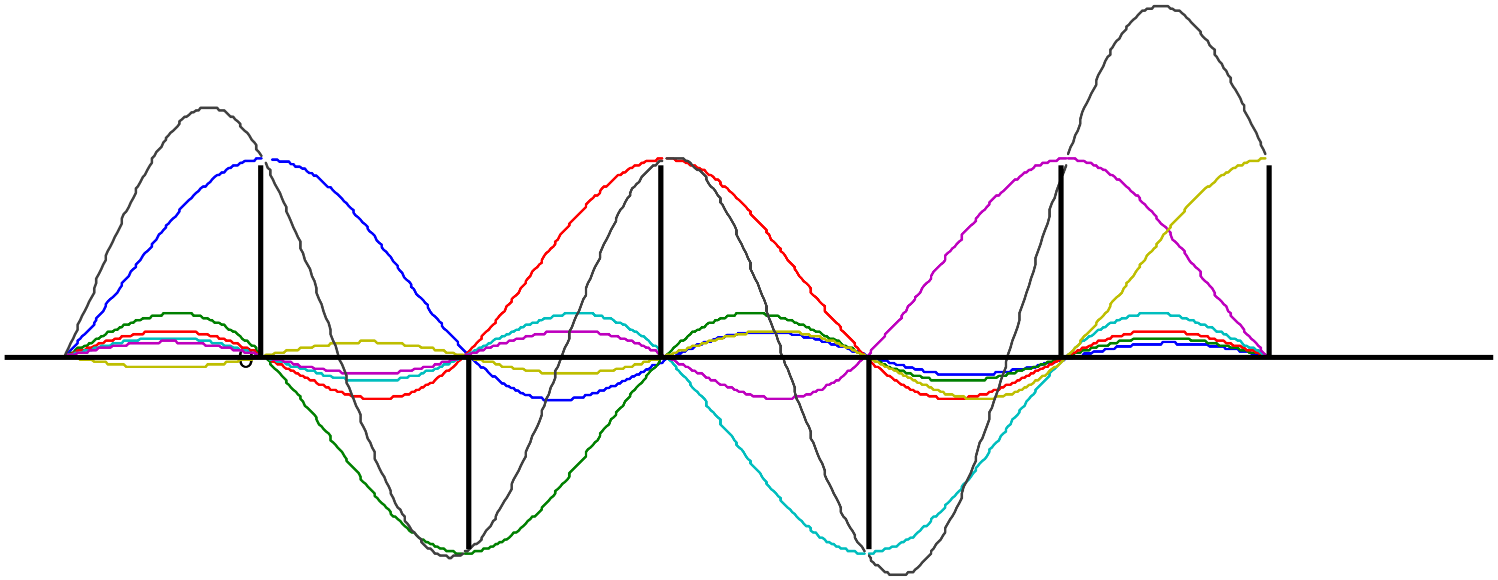


FOR NEXT WEEK

- Read:
  - Chapter 5: §5.11
  - Chapter 6: §6.1-6.10, 6.12-6.17 (<u>not</u> 6.14.2)

- Solve problems:
  - Chapter 5: 5.1, 5.5, 5.6, 5.12

Inter-symbol interference



Inter-symbol interference