Pricing in computer networks
Cost-based pricing

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Slides based on: Courcoubetis and Weber, Pricing Communication Networks, Wiley 2003, chap. 7

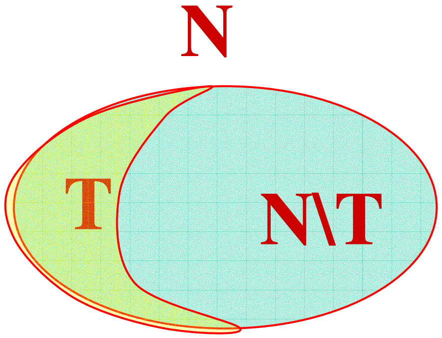
Foundations of cost-based pricing

- welfare maximization is not the entire story: **profitability**! customer satisfaction! (deter new entrants capturing some customers)
- prices based on cost, fair and stable under potential competition (subsidy-free, sustainable)
- allocate cost when factory produces jointly *more* goods, consumers may form *coalitions*, or they may buy services from both monopolist and new entrant
- practical schemes:
  - FDC Fully Distributed Cost – based on accounting records
  - LRIC long-Run Incremental Cost – bottom up, based on optimized models
- in general solution is not unique!
- flat rate pricing: effects on the market

Fair charges

- fairness: no customer feels he is subsidizing others
- N set of n customers, T subset of N, $c_1,\ldots,c_n$ charges, c(T) stand-alone cost, sub-additive,

  assume cost coverage $\sum_{i\in N} c_i = c(N).$

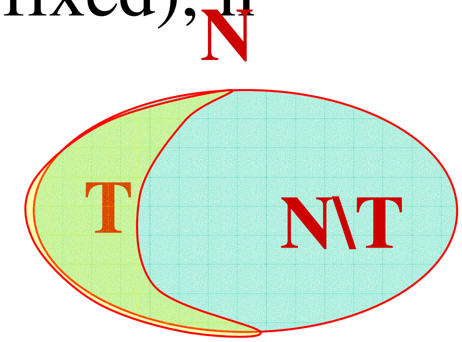


- subsidy-free charges:
  - stand-alone test $$\sum_{i\in T} c_i \leq c(T), \quad \text{for all } T \subseteq N$$
  - incremental cost test $$\sum_{i\in T} c_i \geq c(N) - c(N \setminus T), \quad \text{for all } T \subseteq N$$

    N\T is subsidizing T
- if violated a new entrant can lure away T or N\T

Subsidy-free, support and sustainable prices (2)

- if charges are computed from prices *pi* (*xi* fixed), n services



- *p* subsidy-free price:

whole service

  - stand-alone test $$\sum_{i \in T} p_i x_i \leq c(T), \quad \text{for all } T \subseteq N$$
  - incremental cost test $$\sum_{i \in T} p_i x_i \geq c(N) - c(N \setminus T), \quad \text{for all } T \subseteq N$$

N = T

- producer operates at *zero profit*, prices must be *above marginal cost* (T = {i}, small xi)
- subsidy-free prices may **not** exist!
- can be difficult to check in practice (incremental easier if common costs are neglected, stand-alone more difficult)

Subsidy-free prices may not exist

- Suppose three services are produced in unit quantities with a symmetric subadditive cost function
- $c(\{i\}) = 2.5$, $c(\{i, j\}) = 3.5$, $c(\{i, j, k\}) = 5.5$, where i, j, k are distinct members of $\{1, 2, 3\}$.
- must have $2 \le pi \le 2.5$, for $i = 1, 2, 3$,

  but also $p1 + p2 + p3 = 5.5$.
- So there are no subsidy-free prices.
- economies of scope are not increasing

  $c(\{i, j, k\}) - c(\{i, j\}) > c(\{i, j\}) - c(\{i\})$.

Subsidy-free, support and sustainable prices

- customer consumes a small fraction of xi, let us consider coalitions where parts of the services are produced. $c(x)$ cost for producing quantities $(x1,...,xn)$
- p is a support price for c at x if:

$$\sum_{i \in N} p_i y_i \leq c(y) \,, \quad \text{for all } y \leq x$$

cannot produce some of the demand for less than it is sold.

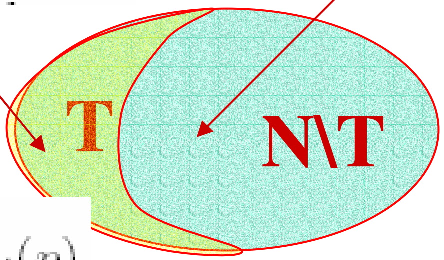
$$\sum_{i \in N} p_i z_i \geq c(x) - c(x - z) \,, \quad \text{for all } z \leq x$$

- are support prices achievable in a market, where demand is function of price? If yes (x is the vector demanded at support price p): anonymously equitable prices

If prices affect demand

- assume a **new service reduces demand for an old service**, to cover cost, prices of old service must increase → customers of old service feel that they are subsidizing

$p'_i = \infty,\ i \in T$ $\quad p'_i = p_i,\ i \notin T$



- p' be the initial situation (not sell any service in T)
  - demand reduction $$\sum_{i\in N\setminus T} p'_i x_i(p') > \sum_{i\in N\setminus T} \underset{\uparrow}{p_i} x_i(p)$$
  - c( ) can be such that $$\sum_{i\in T} p_i x_i(p) \underset{\uparrow}{>} c(x(p)) - c(x(p')) \underset{\uparrow}{>} \sum_{i\in N} p_i x_i(p) - \sum_{i\in N\setminus T} p'_i x_i(p')$$

incremental cost test OK

net incremental revenue does not cover additional cost

If prices affect demand (2)

- **potential competition**: sustainable prices
- incumbent sets prices to cover cost $\sum_{i \in N} p_i x_i(p) \geq c(x(p))$
- competitor (same cost function) post prices *p'* (less for at least one service), resulting demand $x^E(p, p')$
- sustainable prices: potential entrant cannot post prices less than the incumbent's for some services and serve all or part of the demand without incurring loss.
- no *p'* and *x'* such that

$$\sum_{i \in N} p'_i x'_i \geq c(x') \,, \quad \text{and } p'_i < p_i \text{ for some } i \,, \quad \text{and } x' \leq x^E(p, p')$$

- sustainable prices in contestable markets (hit-and-run entry/exit possible – no time for incumbent to react)
- sustainable prices discuorage *inefficient* entry

necessary:
zero profit
natural monopoly
subsidy-free

Ramsey prices: sustainable?

- Ramsey: max social welfare but recover cost
- not sustainable if any service is prices below marginal cost and economies of scale (if services independent, prices > marginal cost)

$$x_1 p_1 < x_1 \partial c/\partial x_1 \underset{\nearrow \text{concavity}}{<} c(x) - c((0, x_2, \ldots, x_n)).$$

- Ramsey *may* be sustainable if all service priced above marginal cost and economies of scope great enough

Shapley value

- simple model: share cost among *n* customers
- charging algorithm: a vector function φ which divides c(N) as (c1, . . . , cn) = φ1(N), . . . , φn(N)
- Suppose that T ⊆ N and i, j are distinct members of T

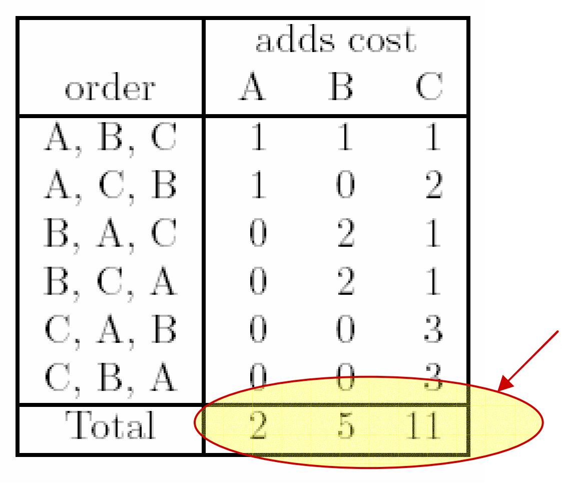
>0… complain unless
<0…

$$\phi_i(T) - \phi_i(T \setminus \{j\}) = \phi_j(T) - \phi_j(T \setminus \{i\})$$

- Fair treatment when new customer is added (assume one is charged more or less, complain unless symmetric situation)
- give costs, there is only one function φ: the Shapley value

Shapley value (2)

- Shapley value for player *i*: expected incremental cost of providing his service when provision of the services accumulates in random order (charge depends on the incrementalcost for which he is reponsible)
- Ex. sharing the cost of a runway (A,B,C require 1,2,3 km)



	adds cost		
order	A	B	C
A, B, C	1	1	1
A, C, B	1	0	2
B, A, C	0	2	1
B, C, A	0	2	1
C, A, B	0	0	3
C, B, A	0	0	3
Total	2	5	11

payment proportional to this

- Shapley does not necessarily satisfy stand-alone and incremental cost tests

Nucleolus *c* of the coalitional game

- other story for "fair" allocation
- c an imputation of cost $\sum_{i \in N} c_i = c(N) \quad \text{and} \quad c_i \leq c(\{i\}), \text{ for all } i$

- the (unique) nucleolus is *c* such that for all *c'* and subsets T s.t.

$$\sum_{i \in T} c'_i < \sum_{i \in T} c_i$$

increment over stand-alone cost

there is subset U s.t. $\sum_{i \in U} c'_i > \sum_{i \in U} c_i \quad \text{and} \quad \sum_{i \in U} c'_i - c(U) > \sum_{i \in T} c_i - c(T)$

if T prefers *c'* then U can object

- Note: "fair allocation" is not uniquely defined, choice depends on unfairness we are trying to avoid!

Second-best core *pi*

- consider benefits when allocating costs
- assume any subset of customers S is free to bypass a monopolist
- *p* (price vector) second-best core if no subset S can choose *p'* s.t. they cover cost of demand at *p'* and net benefit is at least equal

$$\sum_{i\in N}\sum_j p_j x_j^i(p) \geq c\left(\sum_{i\in N} x^i(p)\right)$$

and there is no $S \subset N$, and p' such that both

$$\sum_{i\in S}\sum_j p'_j x_j^i(p') \geq c\left(\sum_{i\in S} x^i(p')\right), \text{ and}$$

$$u_i(x^i(p')) - \sum_j p'_j x_j^i(p') \geq u_i(x^i(p)) - \sum_j p_j x_j^i(p), \quad \text{for all } i \in S.$$

- second-best core prices are Ramsey (but Ramsey prices for N may be unstable for a smaller coalition)
- note that customer is **not** allowed to split purchases (sustainable prices)

...a number of criteria to measure whether a proposed set of costs is judged as fair and presents no incentive for bypass or self-supply

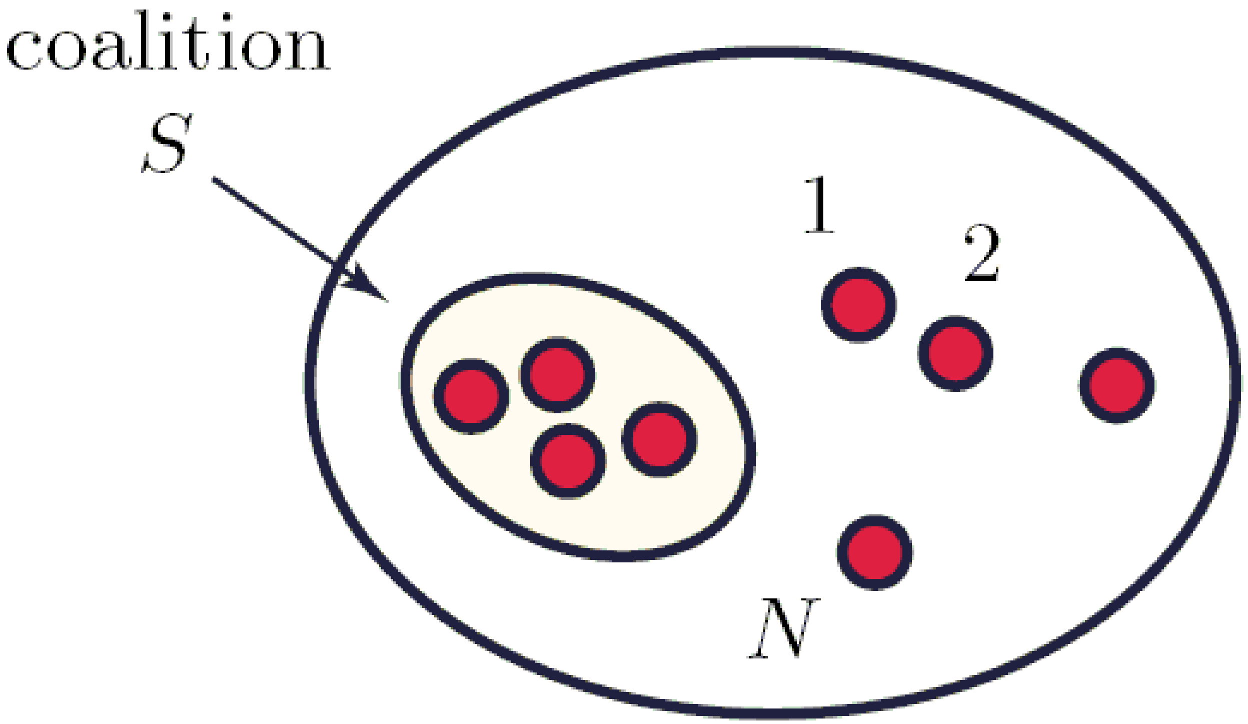


Figure 7.1 The second-best core. The monopolist fixes p s.t. $p^T x - c(x) \ge 0$,where *x* is the aggregate demand, $x = \sum_{i=1}^{N} x^i(p)$ and *c(x)* is the cost of producing *x*. The entrant targets a subset of customers *S* who he wishes to woo. He chooses p^S s.t.$(p^S)^T x^S - c(x^S) \ge 0$ where $x^S = \sum_{i \in S} x^i(p^S)$, and such that the incentive compatibility condition holds, $CS_i(p_i^S) \ge CS_i(p)$, for all $i \in S$.

p is in the second-best core if an entrant has no such possibility.

Nash bargaining game

- cost-sharing: let customer bargain
- $x = (x_{ij})$ quantity of service j supplied to customer i paying cost c_i
- possible allocations of output and cost as y ∈ Y , where y = (x, c1, . . . , cn), with Σi ci = c(x)
- customer has utility $u_i(y)$, they bargain to determine a point u in the bargaining set U = {(u_1(y), . . . , un(y)) : y ∈ Y }
- procrastination is penalized $\exp(-(n - 1)\overset{\overset{\text{seconds of each round}}{\downarrow}}{s}\, \eta i).$
- two players: 1 proposes (u1, u2), 2 prop. (v1, v2)

Nash bargaining game (2)

- no point to make a proposal that will not be accepted $u_2 = e^{-s\eta_2} v_2, \quad \text{and} \quad v_1 = e^{-s\eta_1} u_1$

$$u_1^{1/\eta_1} u_2^{1/\eta_2} = v_1^{1/\eta_1} v_2^{1/\eta_2}$$

- point must lie on a curve where $u_1^{1/\eta_1} u_2^{1/\eta_2}$ is constant and maximized

$$w_1 \log u_1 + w_2 \log u_2. \qquad w_i = 1/\eta_i,$$

- weighted proportional fairness
- Nash bargaining solution (ηi equal) and d utility without bargaining

$$\text{maximize} \prod_{i=1}^{N} (u_i - d_i)$$

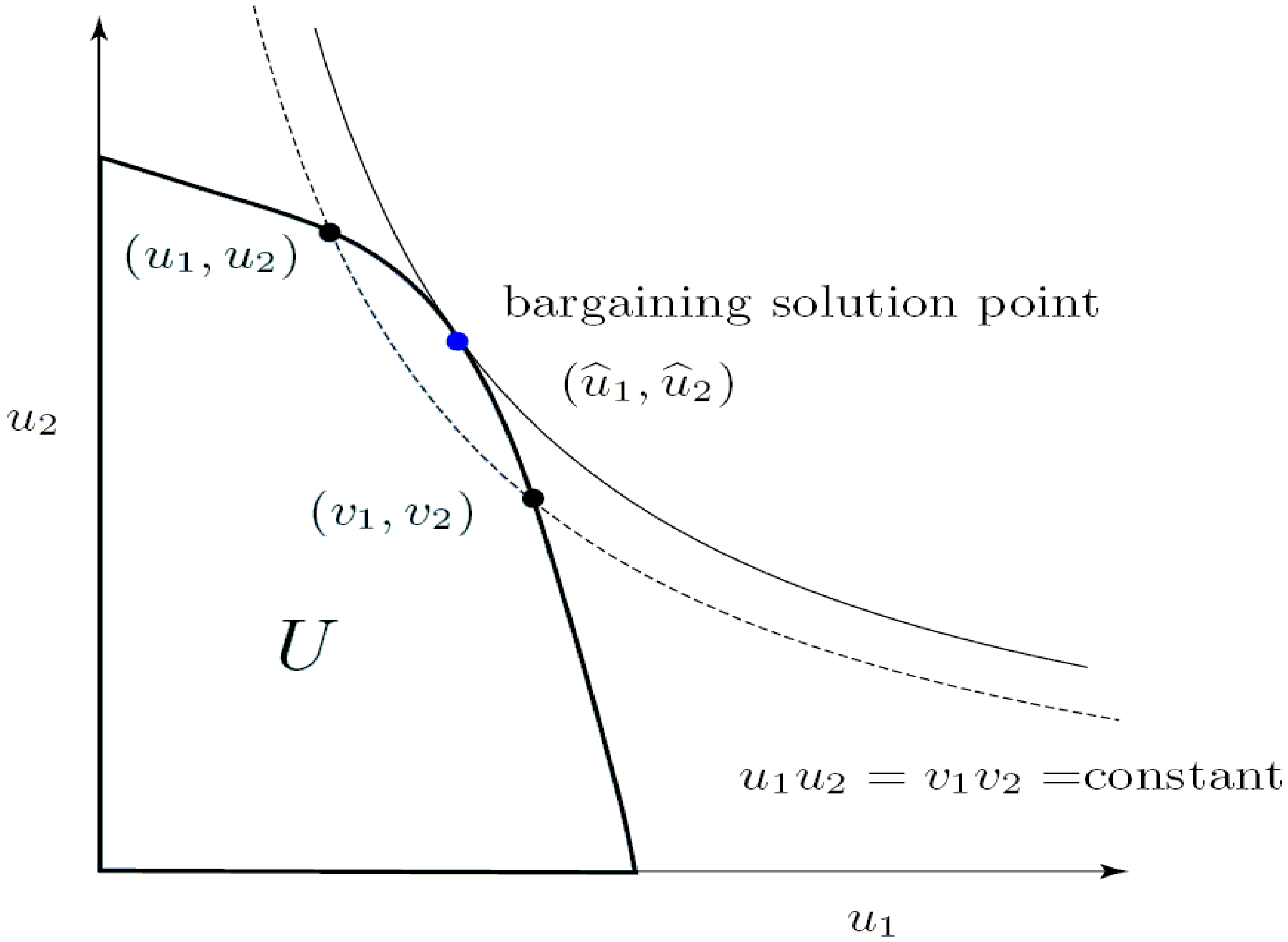


Figure 7.2 Nash's Bargaining game. Two players of equal bargaining power are to settle on a point in *U*. The Nash bargaining solution is at the point in *U* where the product u_1u_2 is maximized.

Pricing in practice

- difficult to know the cost function, bundles, common cost... *cost causation, objectivity, transparency*
- top-down approach: start with existing cost structure and allocate to products → FDC (e.g. activity based costing)
- bottom-up: compute costs from a model of the most efficient facility → LRIC+

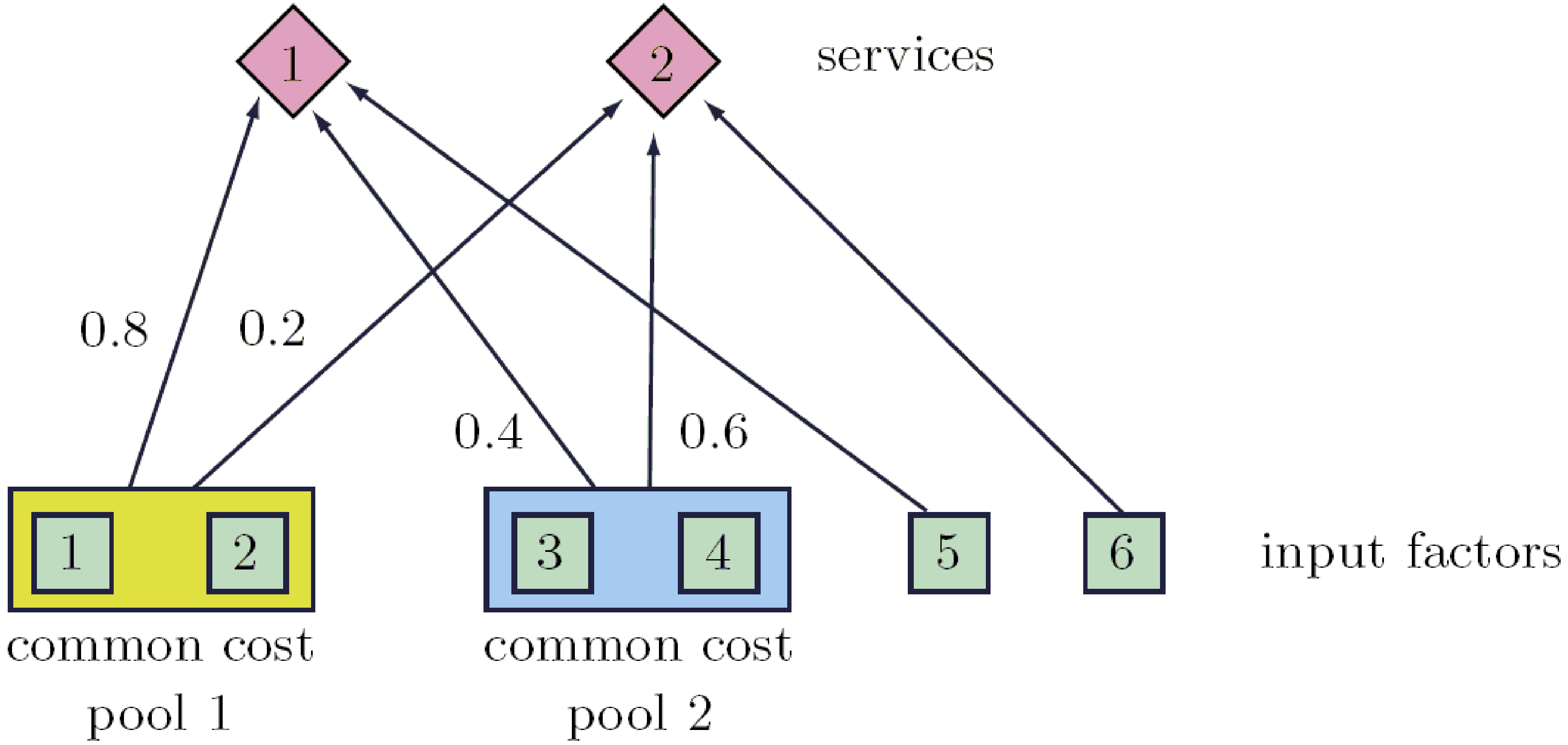


Figure 7.3 In the FDC approach the cost of input factors are assigned to services. For example, service 1 is assigned 0.8 of the cost factors in the first cost pool and 0.4 of the cost factors in the second cost pool. The different common cost pools and the coefficients for sharing the cost of the common factors are defined by the designer of the system.

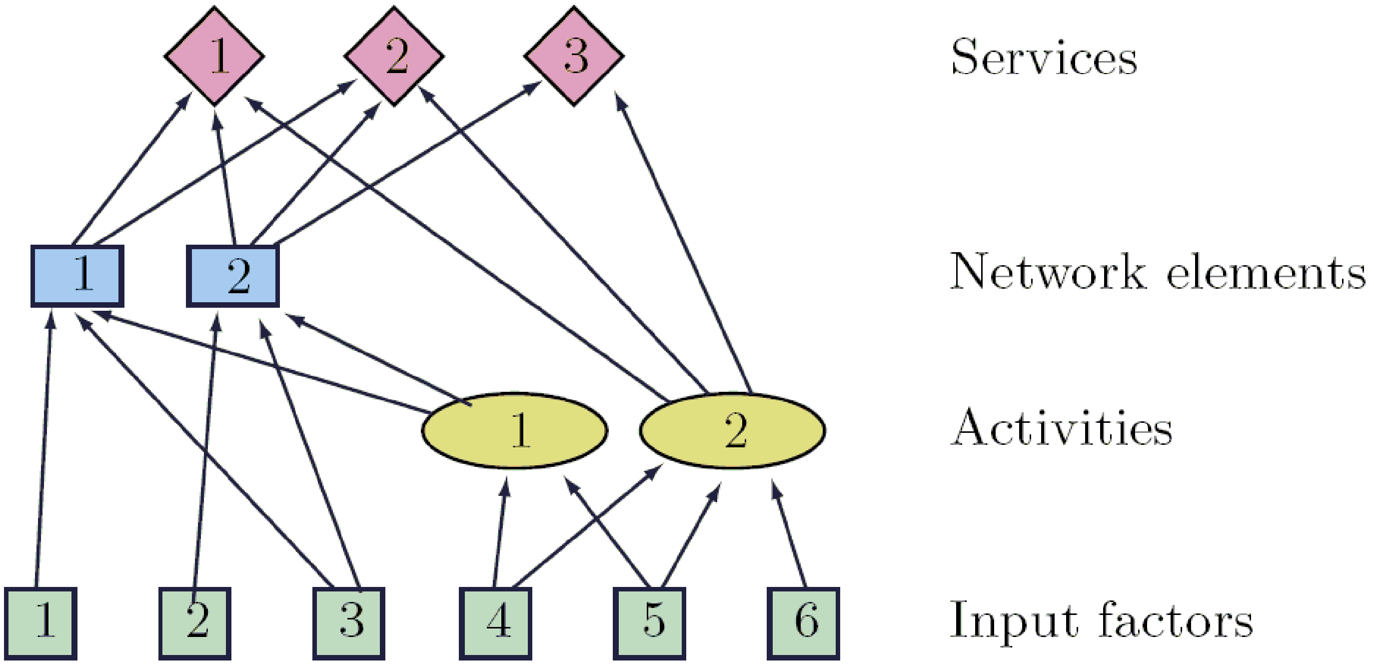


Figure 7.4 The cost of input factors can be assigned to services in a hierarchical fashion. The lowest level are input factors that are consumed by the network operator, such as labour and depreciation of network elements. The next level consists of labour-intensive activities. The next level consists of the network elements such as the routers, switches and links. The last level consists of services. Input cost factors are allocated to network elements and activities. Activities (activity costs) are allocated to network elements or directly to services. The cost of network elements is allocated to each service in proportion to its use by that service. Usually the cost of a service also includes the cost of capital it employs. A crucial decision, besides the definition of the activities, is the definition of the coefficients to apportion the costs of one level to the next level up.

Long-run incremental cost (LRIC+)

- subsidy-free, close to the prices in a contestable market, promotes efficient forward-looking investment decisions
- a firm that offers quantities y1 and y2 of services 1 and 2, with cost c(y1, y2).
- LRIC for service 1 is LRIC(y1) = c(y1, y2) - c(y2), where c(y2) is defined for a facility **optimized** to produce only type 2 service
- because of economy of scope LRIC(y1) ≤ SAC(y1) – stand-alone cost

Long-run incremental cost (2)

- sum of the prices constructed according to LRIC will not in general cover the production cost.

$$\begin{aligned}\mathrm{LRIC}(y_1) + \mathrm{LRIC}(y_2) &= c(y_1, y_2) + [c(y_1, y_2) - c(y_1) - c(y_2)] \\ &\leq c(y_1, y_2)\,,\end{aligned}$$

- some common fixed cost is not recovered: distribute common cost such that

stand alone cost

$$\mathrm{LRIC}(y_i) \leq p(y_i) \leq \mathrm{SAC}(y_i)$$

and sum of prices equal total cost (LRIC+)

- construct prices that would prevail in a competitive market (use current rather than historic cost)

Efficient component pricing

- alternative to LRIC+ that considers the incumbent’s opportunity cost
- e.g. for unbundling of the local loop

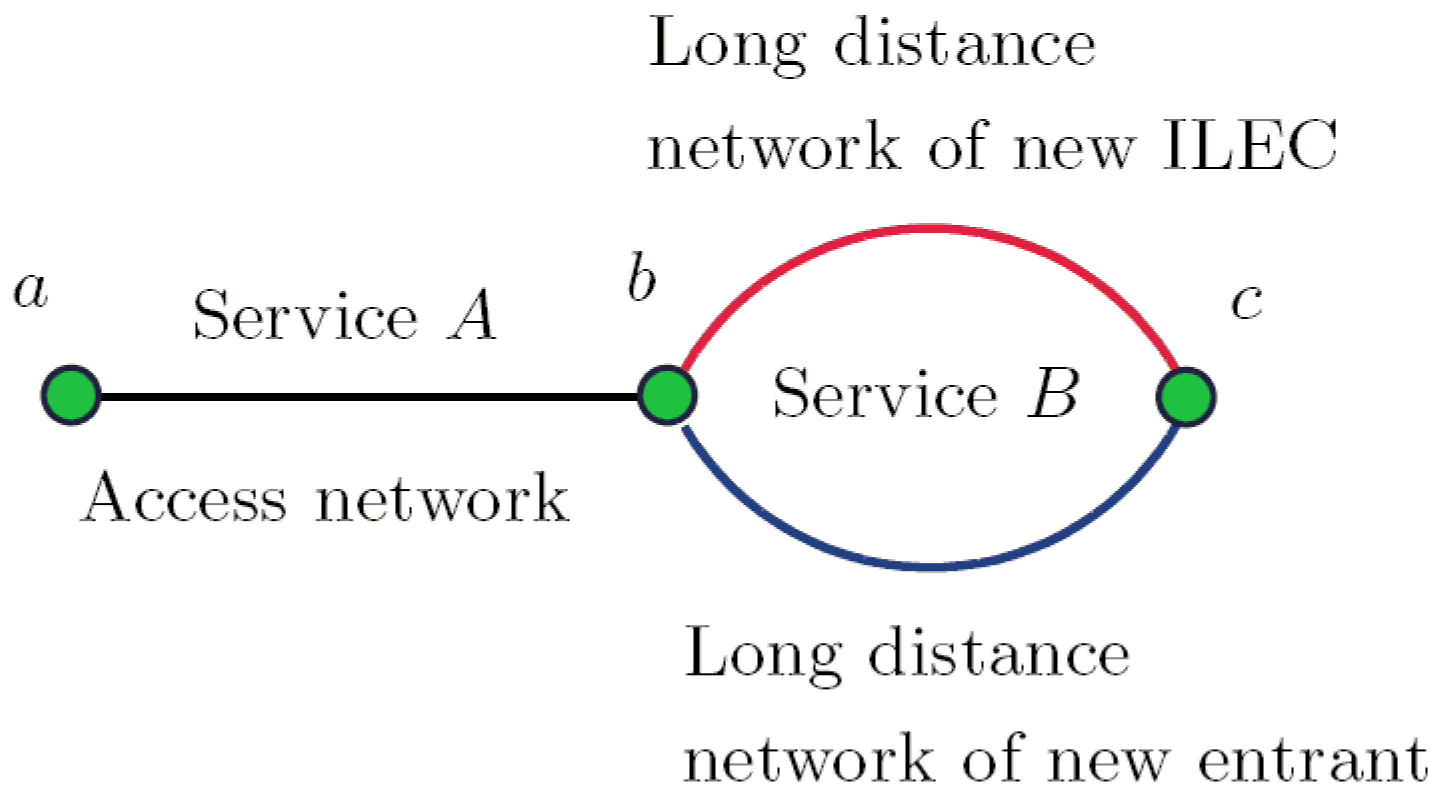


Figure 7.5 The Efficient Component Pricing Rule for pricing network services. Service *A* connects a to b; service *B* connects *b* to *c* and service *AB* connects a to c. According to the *ECPR* an incumbent should charge for *A* a rental price of $p_A = p_{AB} - c_B = c_A + (p_{AB} - c_A - c_B)$ where p_i, c_i are the price and cost of providing a unit of service *i*. Note that p_A is the cost of service *A* plus the private opportunity cost to the incumbent of not offering a unit of service *AB*.

Efficient component pricing

- deters inefficient entrants but
    1. reduces profit of entrant (inefficient incumbent → tax on new entrants)
    2. guarantees incumbent’s profit margin (even if inefficient)
    3. no motivation for accurate cost estimation (historic cost)
    4. incumbent has no incentive to reduce c_A, while C_B reduction will increase rental price of A
    5. incumbent can increase market share in provisioning AB
    6. Administrative problems (same element can be rented at different prices depending on the service)

Comparing FDC, LRIC

- FDC (with historic costs) advantages:
  - easier to develop since (accounting)
  - easy to audit by regulators.
- disadvantages:
  - no incentives for improving the efficiency and deploying newer technologies
  - not always based on causal relations but depends on arbitrarily chosen coefficients (problem reduced if one uses the activity-based costing)
- LRIC+ combined with bottom-up models using current costs, advantages:
  - prices that are subsidy-free, hence stable and economically efficient;
  - does not include inefficiencies that are due to decisions made in the past, and provides the right competitive signals
- disadvantages:
  - hard to develop due to the complexity of the bottom-up models
  - accountants find them hard to understand.

Flat-rate pricing

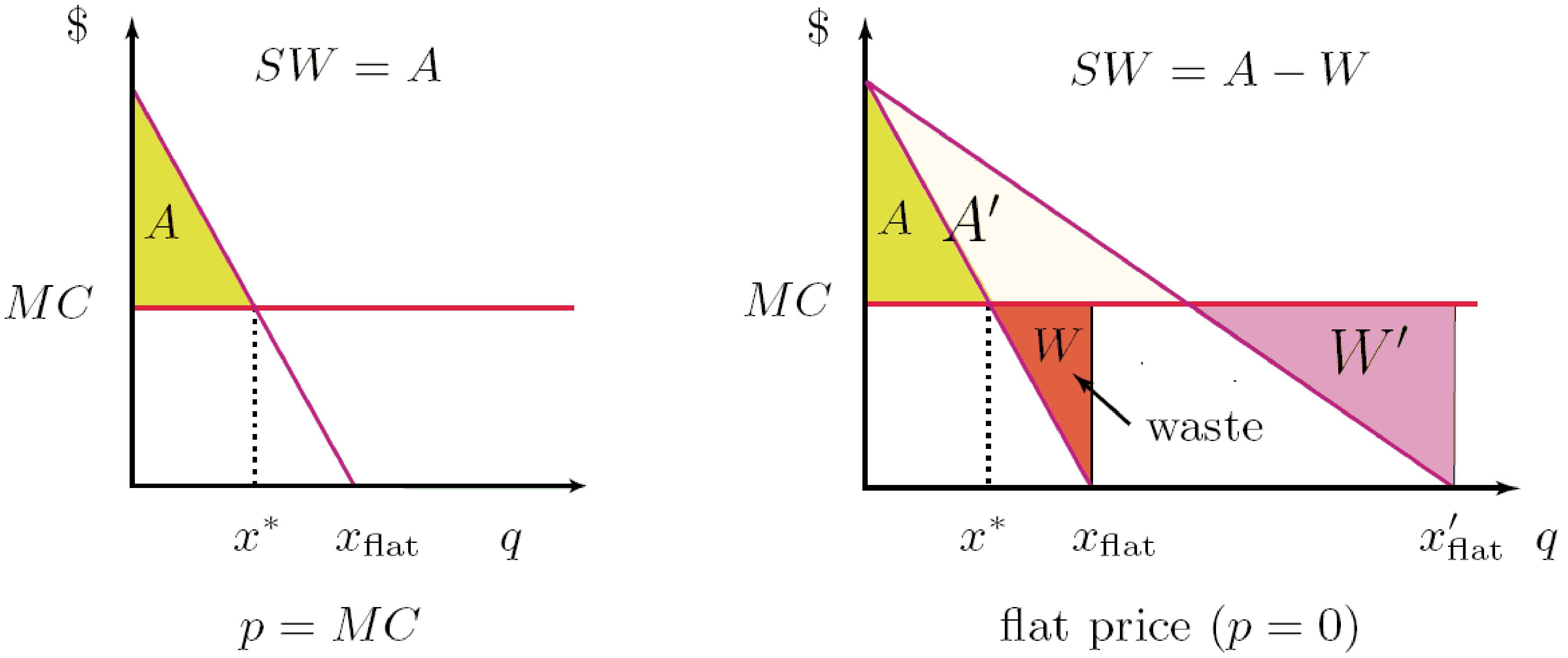
- determined *a priori* (e.g. flat-fee Internet, all-you-can-eat restaurant)
- leads to waste, unstable under competition because light users subsidize heavy users

- bad effects can be reduced by restricting range for resource usage: m contracts, such that the i-th contract limits v to the interval [0, ki], where $0 < k1 < \cdots < km < M$

→ incentive to predict

→ need policing (soft policing?: pay extra for larger amounts)

→ possibility to dynamically switch contract may increase value



$p = MC$

flat price $(p = 0)$

Figure 7.6 Social waste under flat rate pricing. If a user is charged a price $p = MC$ then he consumes x^* and the social welfare is the area *A*. However, if he is charged a flat price, say $p = 0$, then he has no incentive to reduce his consumption and so consumes x_{flat}. This makes the social welfare $A - W$ where *W* is social waste. Thus charging only a flat fee encourages social waste. For a demand function with a greater demand, so the demand at $p = 0$ is x'_{flat}, the social waste of W' is even greater.

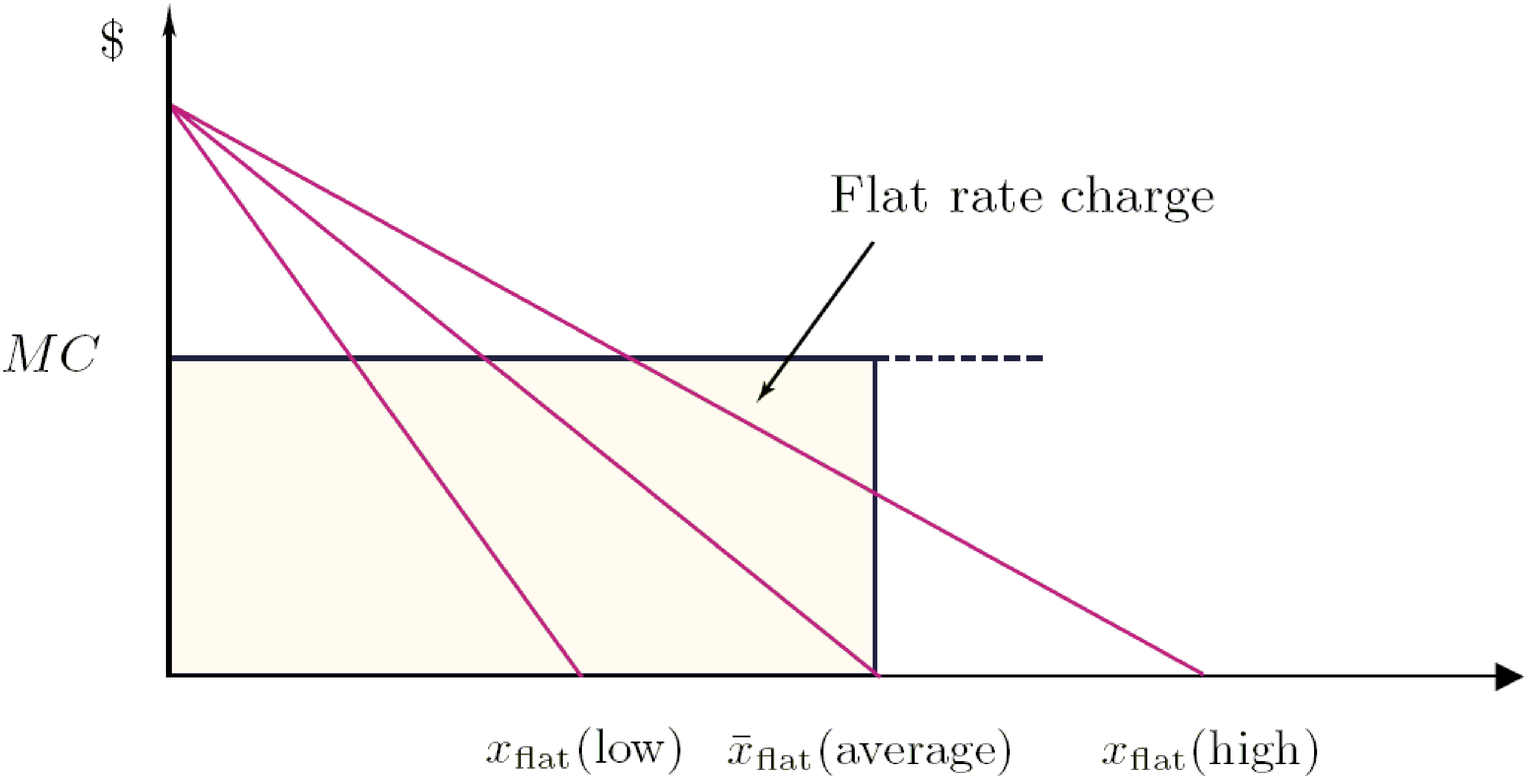


Figure 7.7 Cross subsidization with a flat fee. Suppose a flat fee is charged, sufficient to cover the cost of average usage, i.e., *fee*= $x_{flat}MC$. However, having paid that fee, a low users will consume x_{flat} (low) and find that he has negative net benefit. Given this, he will chose not to buy from this service provider.